



III Webinar 2021

26/08/2021 , 17h30

CT de Identificação de Sistemas e Ciência de Dados



<https://www.youtube.com/watch?v=5bB6h0cFSJQ>

Análise e previsão de séries temporais

Time series analysis and forecasting

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Google Scholar: <https://scholar.google.com/citations?user=0X7VkC4AAAAJ&hl=pt-PT>

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Top H-Index for Scientists in Brazil: <http://www.guide2research.com/scientists/BR>

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Dynamical systems and time series forecasting

PPGEPS-PUCPR (Programa de Pós-graduação em Engenharia de Produção e Sistemas)
PPGEM-PUCPR (Programa de Pós-graduação em Engenharia Mecânica) (collaboration)
PPGEE-UFPR (Programa de Pós-graduação em Engenharia Elétrica)

Hybrid multi-stage decomposition with parametric model applied to wind speed forecasting in Brazilian Northeast

S. R. Moreno, V. C. Mariani, L. S. Coelho

Renewable Energy, <https://www.sciencedirect.com/science/article/abs/pii/S0960148120316980>

Novel hybrid model based on echo state neural network applied to the prediction of stock price return volatility

G. T. Ribeiro, A. A. P. Santos, V. C. Mariani, L. S. Coelho

Expert Systems with Applications, <https://www.sciencedirect.com/science/article/abs/pii/S0957417421009003>

Hybrid wavelet stacking ensemble model for insulators contamination forecasting

S. F. Stefenon, M. H. D. M. Ribeiro, A. Nied, V. C. Mariani, L. S. Coelho, V. R. Q. Leihardt

IEEE Access, <http://dx.doi.org/10.1109/ACCESS.2021.3076410>

Multi-step wind speed forecasting based on hybrid multi-stage decomposition model and long short-term memory neural network

S. R. Moreno, R. G. da Silva, V. C. Mariani, L. S. Coelho

Energy Conversion and Management, <https://www.sciencedirect.com/science/article/abs/pii/S0196890420304076>

Enhanced ensemble structures using wavelet neural networks applied to short-term load forecasting

G. T. Ribeiro, V. C. Mariani, L. S. Coelho

Engineering Applications of Artificial Intelligence, <https://www.sciencedirect.com/science/article/abs/pii/S0952197619300624>

A novel decomposition-ensemble learning framework for multi-step ahead wind energy forecasting

R. G. da Silva, M. H. D. M. Ribeiro, S. R. Moreno, V. C. Mariani, L. S. Coelho

Energy, <https://www.sciencedirect.com/science/article/abs/pii/S0360544220322817>

Multi-step ahead meningitis case forecasting based on decomposition and multi-objective optimization methods

M. H. D. M. Ribeiro, V. C. Mariani, L. S. Coelho

Journal of Biomedical Informatics, <https://www.sciencedirect.com/science/article/abs/pii/S1532046420302033>

Forecasting Brazilian and American COVID-19 cases based on artificial intelligence coupled with climatic exogenous variables

R. G. da Silva, M. H. D. M. Ribeiro, V. C. Mariani, L. S. Coelho

Chaos, Solitons & Fractals, <https://www.sciencedirect.com/science/article/abs/pii/S0960077920304252>

Electrical insulators fault forecasting based on a wavelet neuro-fuzzy system

S. F. Stefenon, R. Z. Freire, L. S. Coelho, L. H. Meyer, R. B. Grenogi, W. G. Buratto, A. Nied

Energies, <http://dx.doi.org/10.3390/en13020484>

Short-term forecasting COVID-19 cumulative confirmed cases: Perspectives for Brazil

M. H. D. M. Ribeiro, R. G. da Silva, V. C. Mariani, L. S. Coelho

Chaos, Solitons & Fractals, <https://www.sciencedirect.com/science/article/abs/pii/S0960077920302538>

An R library for nonlinear black-box system identification

H. V. Ayala, M. C. Gritti, L. S. Coelho

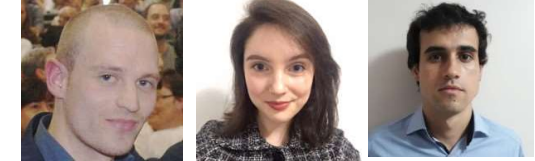
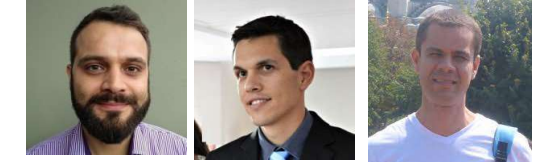
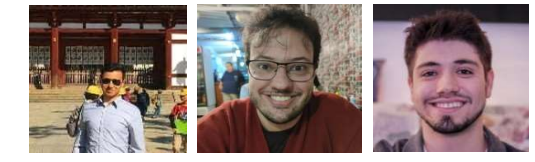
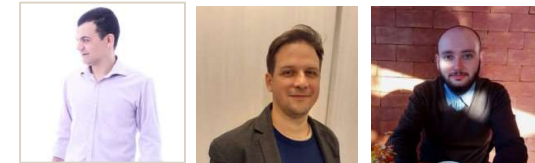
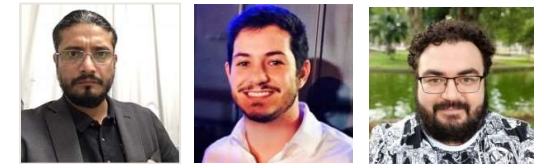
SoftwareX, <http://dx.doi.org/10.1016/j.softx.2020.100495>



2021



2020



Agenda



Time series **fundamentals**



Time series **patterns**



Time series **decomposition**

Time series forecasting



Time series forecasting using **linear** models

Time series forecasting using **nonlinear** models

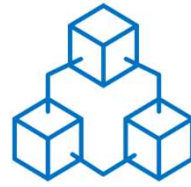
Time series forecasting using **machine learning**



Performance analysis



References



Time series fundamentals



What is a time series?

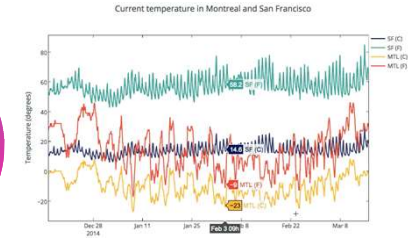
A **time series** is a **sequence of data** points recorded through time (collected at regular intervals).

The **time series analysis** is concerned with the systematic approaches to **extract information** from *time series*, i.e. from observations ordered in time.

The analysis is capable of giving us useful **insights** on how a variable changes over time.

Time series forecasting is the use of statistical methods to **forecast future behavior** based on historical data.

Time
series data



Interactive
visualizations

analysis
search
manipulate
encode

Data
science



Machine
learning

regression

classification

clustering

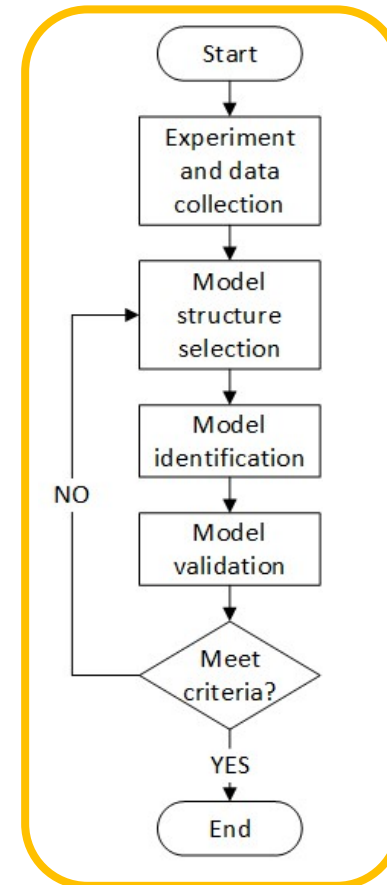
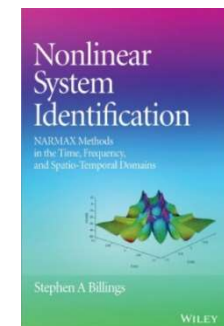
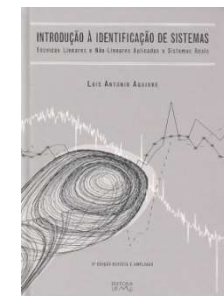
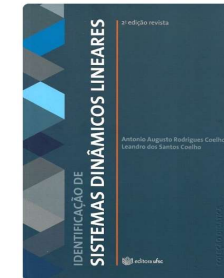
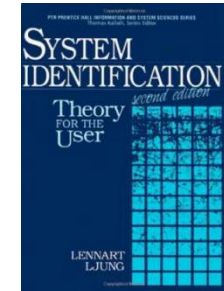
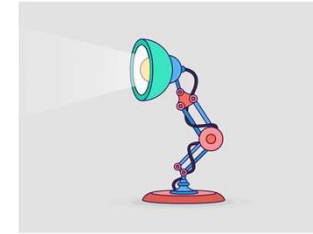
System identification and time series forecasting

System identification is a **data driven modeling** approach related with obtaining a (mathematical) model from or for a time series.

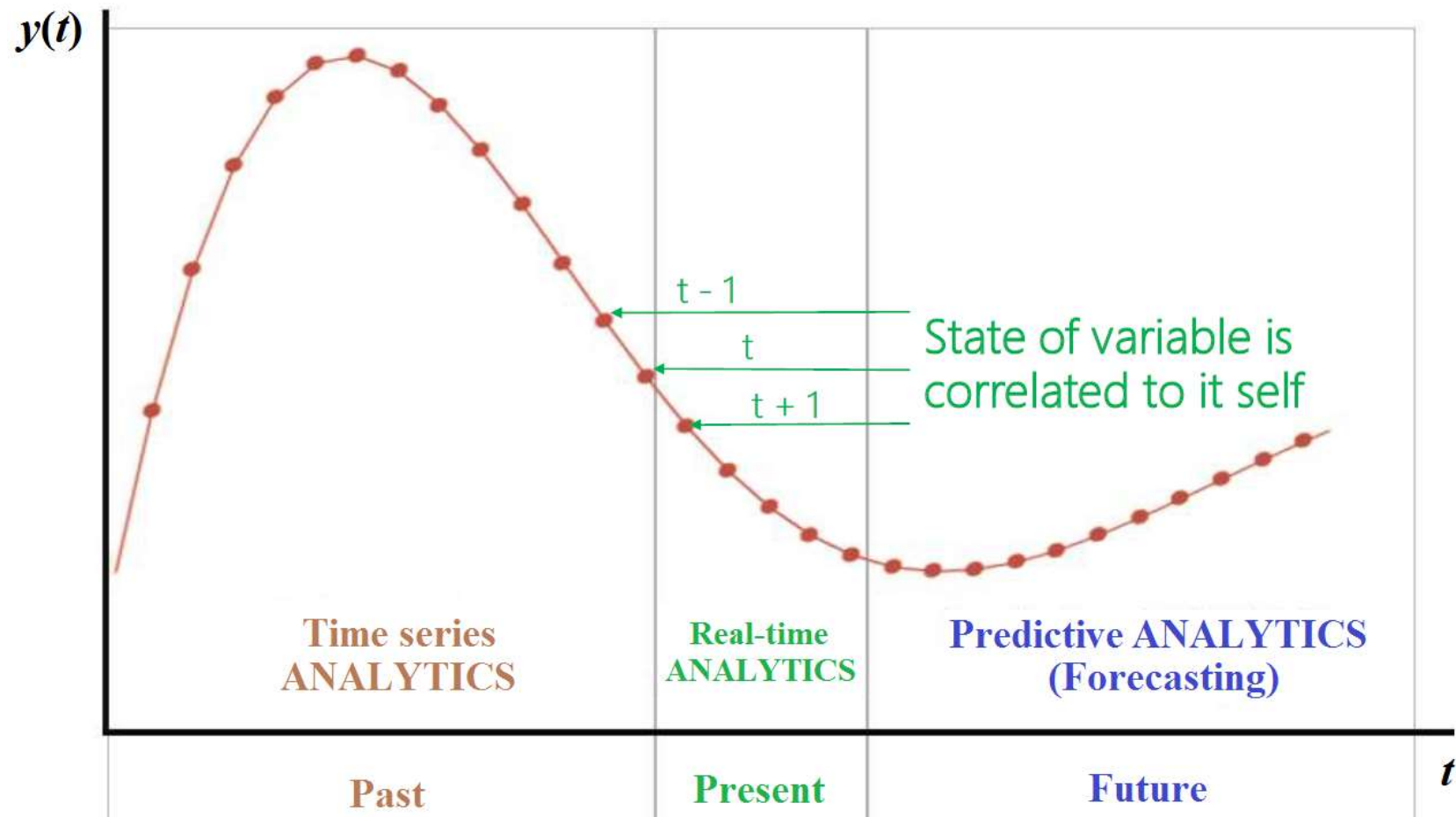
A system identification **procedure** is a set of rules for attaching a system (model) to time series data. In particular such **rules may be algorithms**.

The **core part** of system identification consists of the rules (in particular algorithms) of attaching a “good” model from the model class to the preprocessed data.

Both construction and evaluation of the **quality of such rules** form the center part of the theory and of development of methods of system identification.



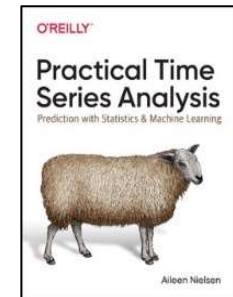
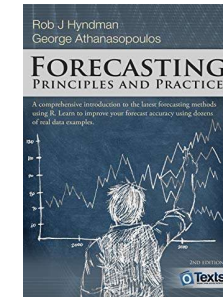
The time series connect past, present and future



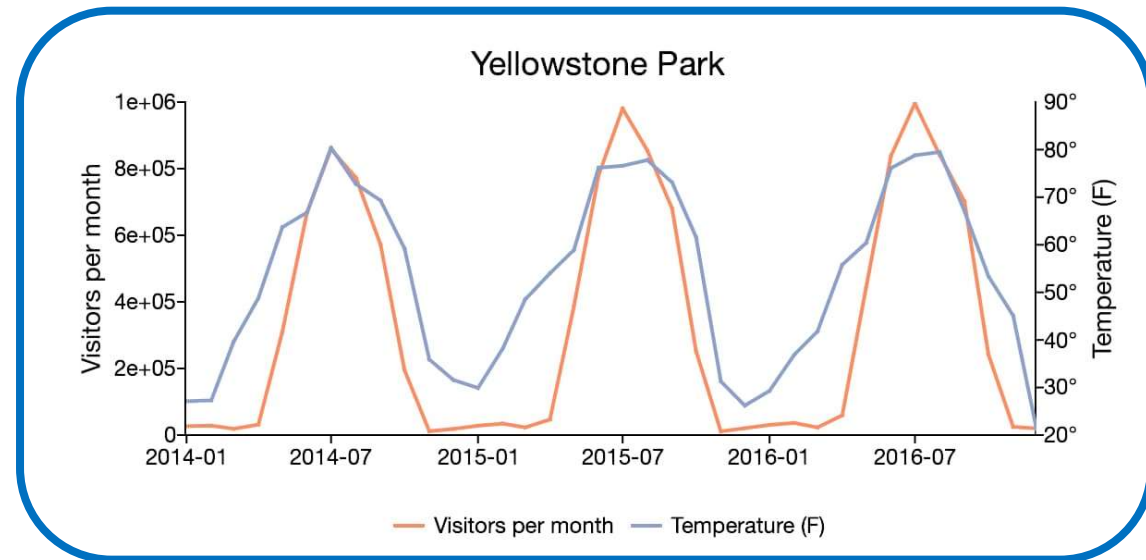
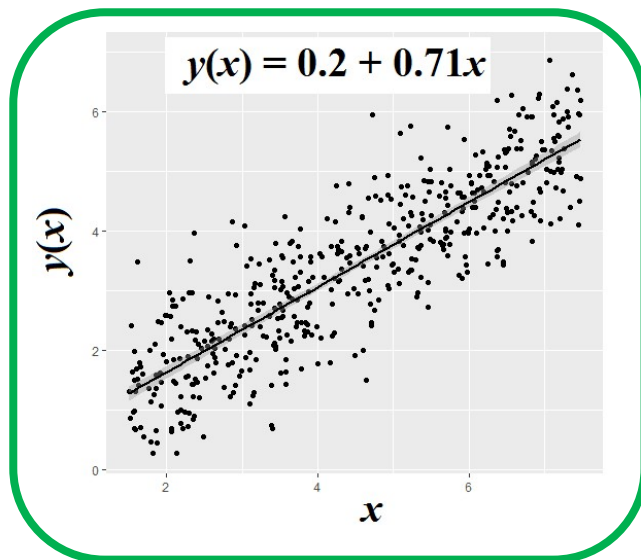
Difference between forecast and prediction?

In **time series**, forecasting seems to mean to estimate a future values given past values of a time series (dynamical systems theory).

In **regression**, prediction seems to mean to estimate a value whether it is future, current or past with respect to the given data.



<https://otexts.com/fpp2/>

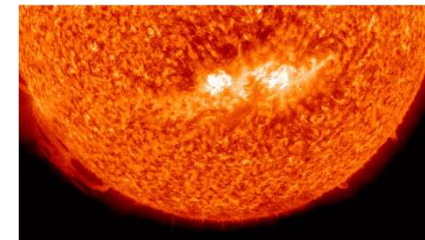
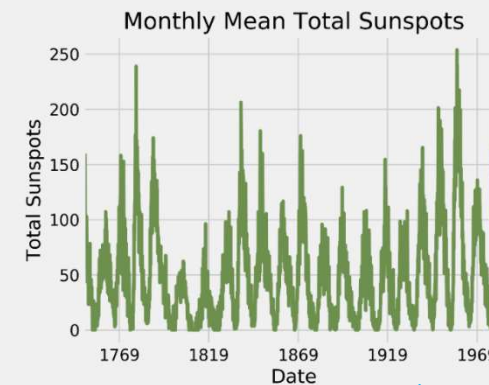
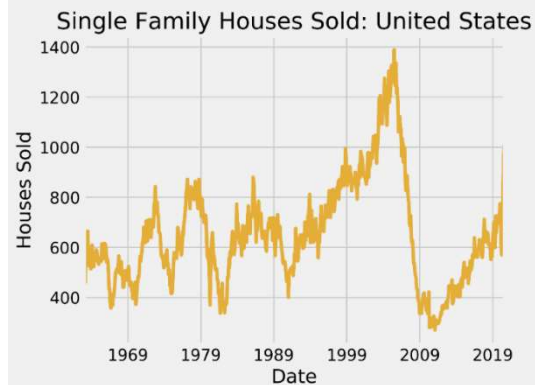
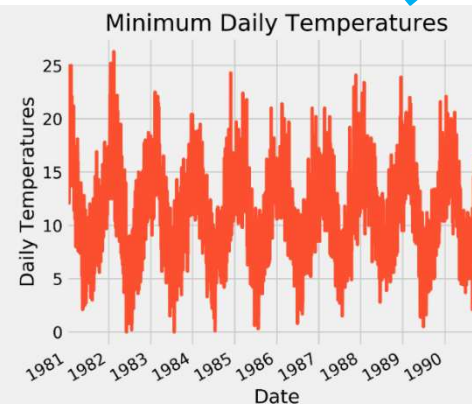
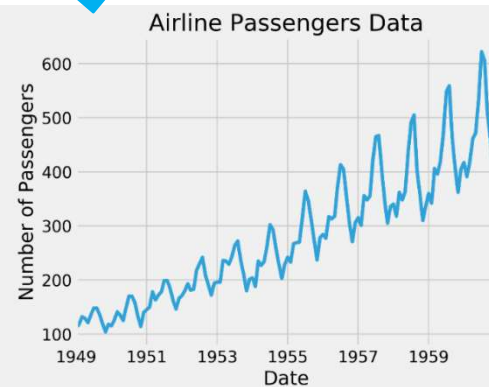


Forecasting would be a subset of prediction (supervised regression). Any time you predict into the future it is a forecast. All forecasts are predictions, but not all predictions are forecasts, as when you would use regression to explain the relationship between two variables.

Time series

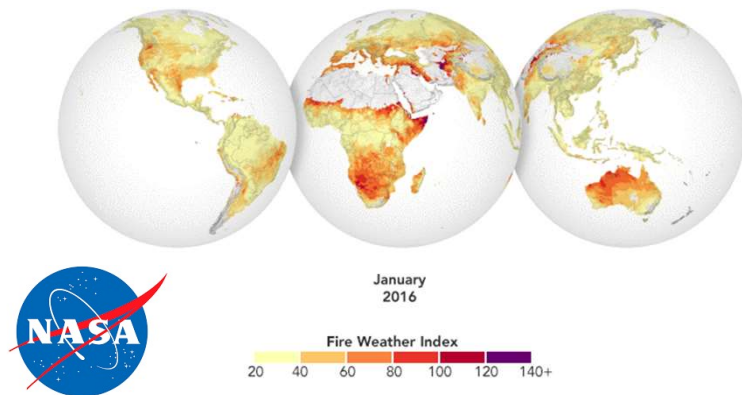


Import libraries and data



Examples of time series include weather data, rainfall measurements, temperature readings, energy data, heart rate monitoring, brain monitoring, quarterly sales, stock prices, automated stock trading, industry forecasts, and interest rates.

Applications related to time series



Forecasting fire

NASA researchers have created a model that analyzes various weather conditions, including rainfall, to predict the formation and spread of fires.

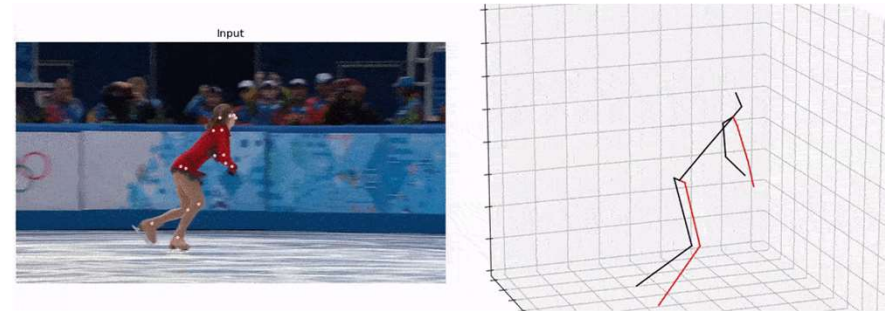
<https://visibleearth.nasa.gov/images/92367/forecasting-fire/92367f>



Stock market forecasting

The time series analysis is a powerful tool for forecasting the trend or even future. The trend chart can provide adequate guidance for the investor.

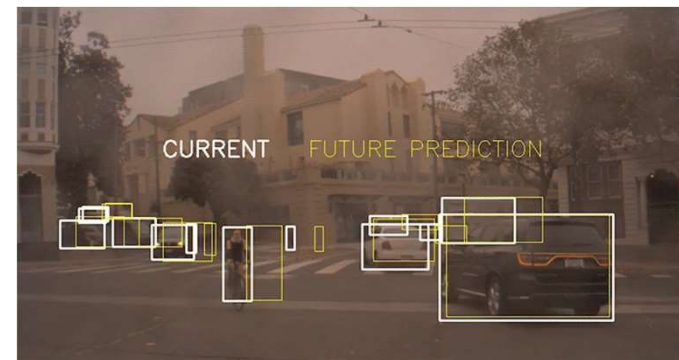
<https://www.theaidream.com/post/stock-market-forecasting-using-time-series-analysis>



3D human pose estimation

The idea of human pose estimation is detecting locations of people's joints, which form a "skeleton".

<https://blog.usejournal.com/3d-human-pose-estimation-ce1259979306>



Autonomous vehicles

The key is to analyze temporal information in an image sequence in a way that generates accurate future motion predictions despite the presence of uncertainty and unpredictability.

<https://blogs.nvidia.com/blog/2019/05/22/drive-labs-predicting-future-motion/>

Types and models of time series analysis

- ✓ **Classification:** Identifies and assigns categories to the data.
- ✓ **Curve fitting:** Plots the data along a curve to study the relationships of variables within the data.
- ✓ **Descriptive analysis:** Identifies patterns in time series data, like trends, cycles, or seasonal variation.
- ✓ **Explanative analysis:** Attempts to understand the data and the relationships within it, as well as cause and effect.
- ✓ **Exploratory analysis:** Highlights the main characteristics of the time series data, usually in a visual format.
- ✓ **Forecasting:** Predicts future data. This type is based on historical trends. It uses the historical data as a model for future data, predicting scenarios that could happen along future plot points.
- ✓ **Intervention analysis:** Studies how an event can change the data.
- ✓ **Clustering/segmentation:** Splits the data into segments to show the underlying properties of the source information.

Aspects that come into play when dealing with time series.

Is it **stationary**? Is there a **seasonality**?

Is the target variable **autocorrelated**?



Analysis



Forecasting

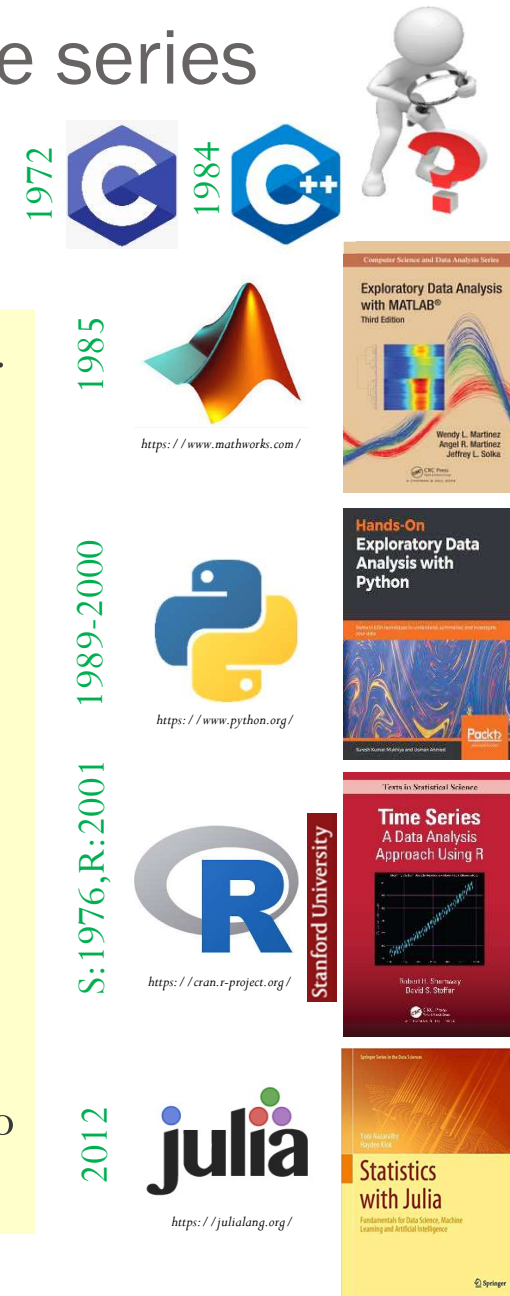
Clustering

Classification

Exploratory data analysis (EDA) for time series

EDA is used by data scientists to analyze and investigate **data** sets and summarize their **main characteristics**.

- 1 Importing required libraries and import **time-series data** sets. Clean the data.
- 2 Get **basic descriptive statistics** and review the summary of time-series data
- 3 Get inference from the **visualization** graphs of time series data
- 4 Check the **behavior** of time series data (correlation plots, stationary check)
- 5 Apply **transformation** functions to convert non-stationary to stationary



Exploratory data analysis (EDA) for time series

Statistics, Probability, and Data science



1962

THE FUTURE OF DATA ANALYSIS

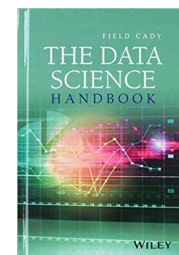
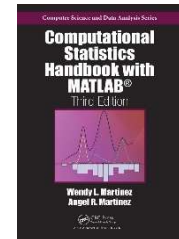
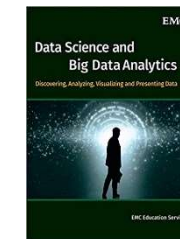
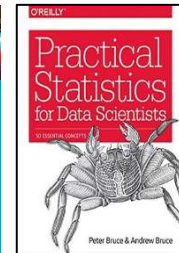
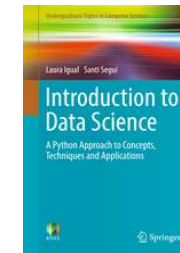
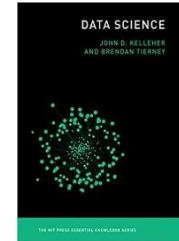
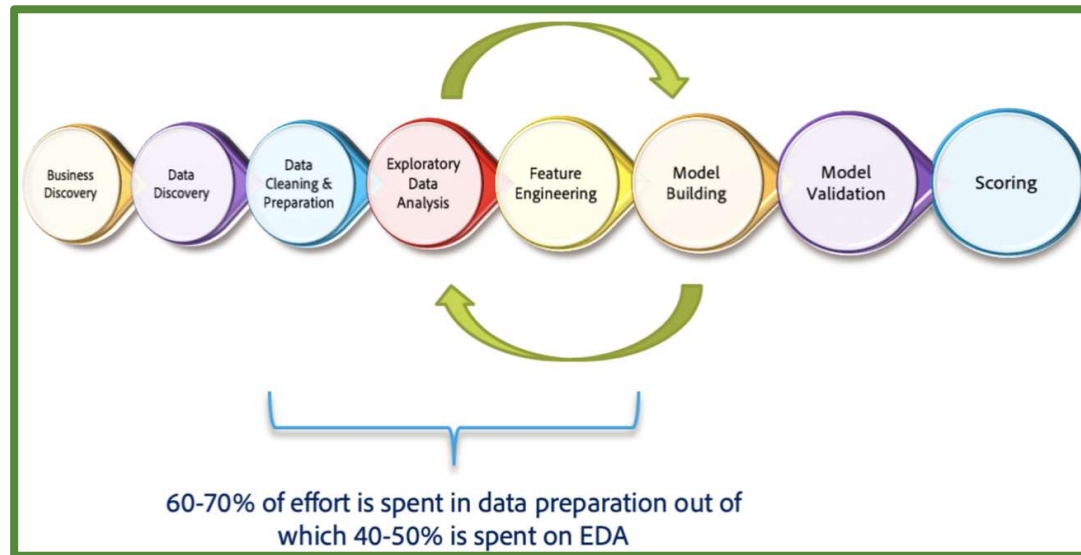
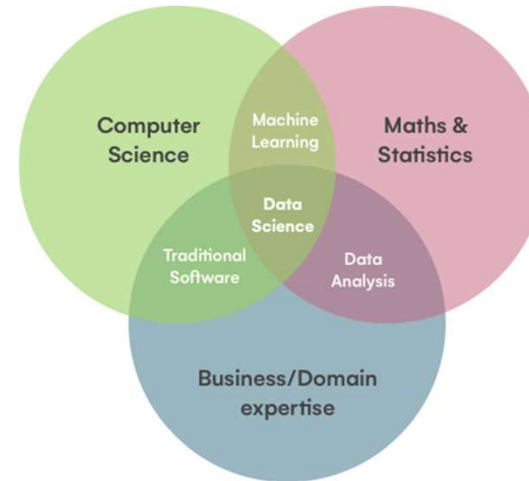
BY JOHN W. TUKEY



PRINCETON
UNIVERSITY



John Tukey
(1915-2000)



Exploratory data analysis (EDA) for time series

Data visualization



Ethiopian commodity market

14

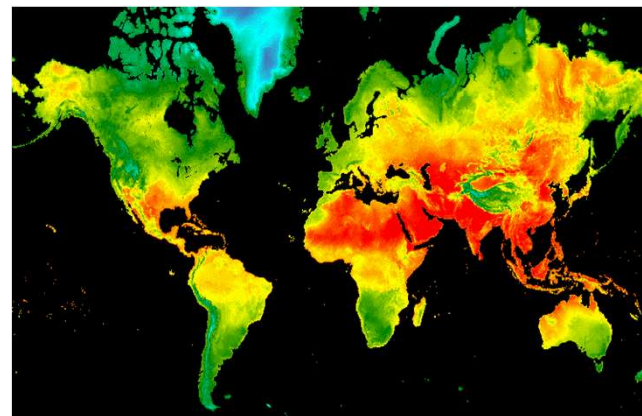
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<https://ads9rca.wordpress.com/2017/01/10/anonymous-analytics/>

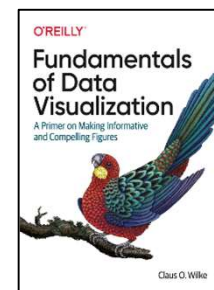
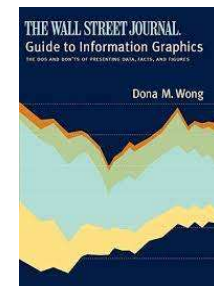
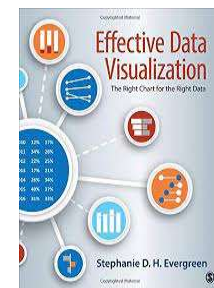
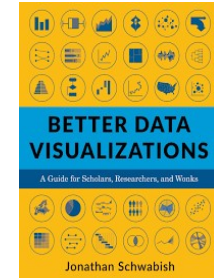
https://www.researchgate.net/publication/268396263_Visualizing_the_Ethiopian_Commodity_Market/figures?lo=1&utm_source=google&utm_medium=organic



Uber



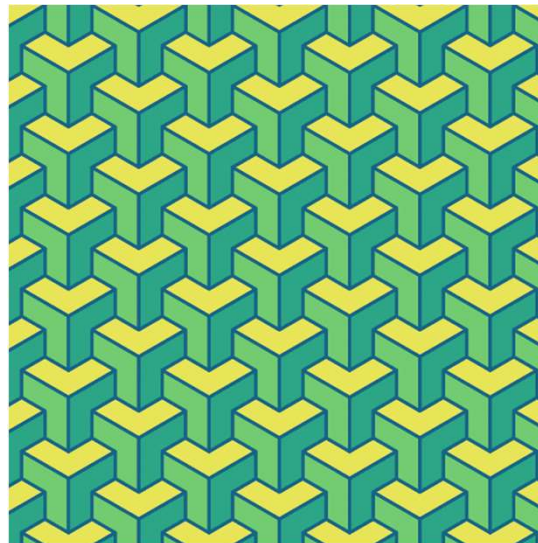
Google Earth Engine



Leandro dos Santos Coelho



Time series patterns



Time series patterns (components)

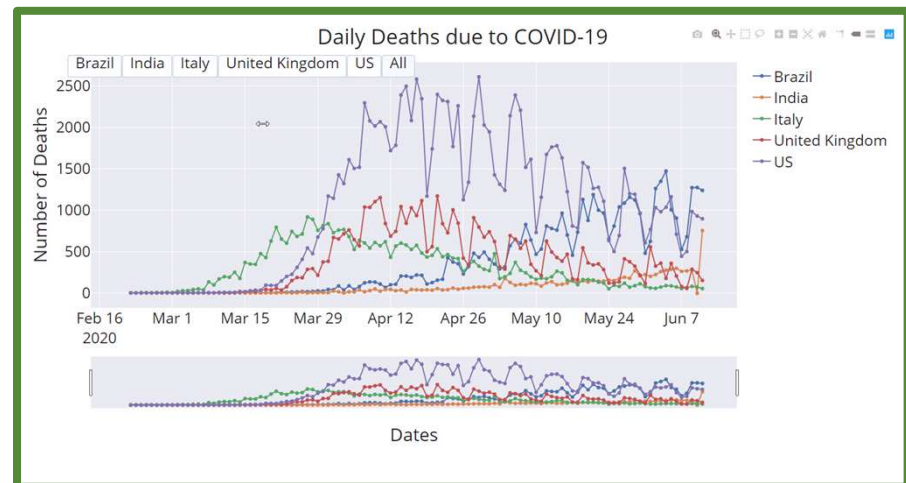
A useful abstraction for selecting forecasting methods is to break a time series down into systematic and unsystematic components.

Systematic: Components of the time series that have **consistency** or **recurrence** and can be **described** and **modeled**.

Non-systematic: Components of the time series that **cannot** be directly modeled.

A given time series is thought to consist of **systematic components**

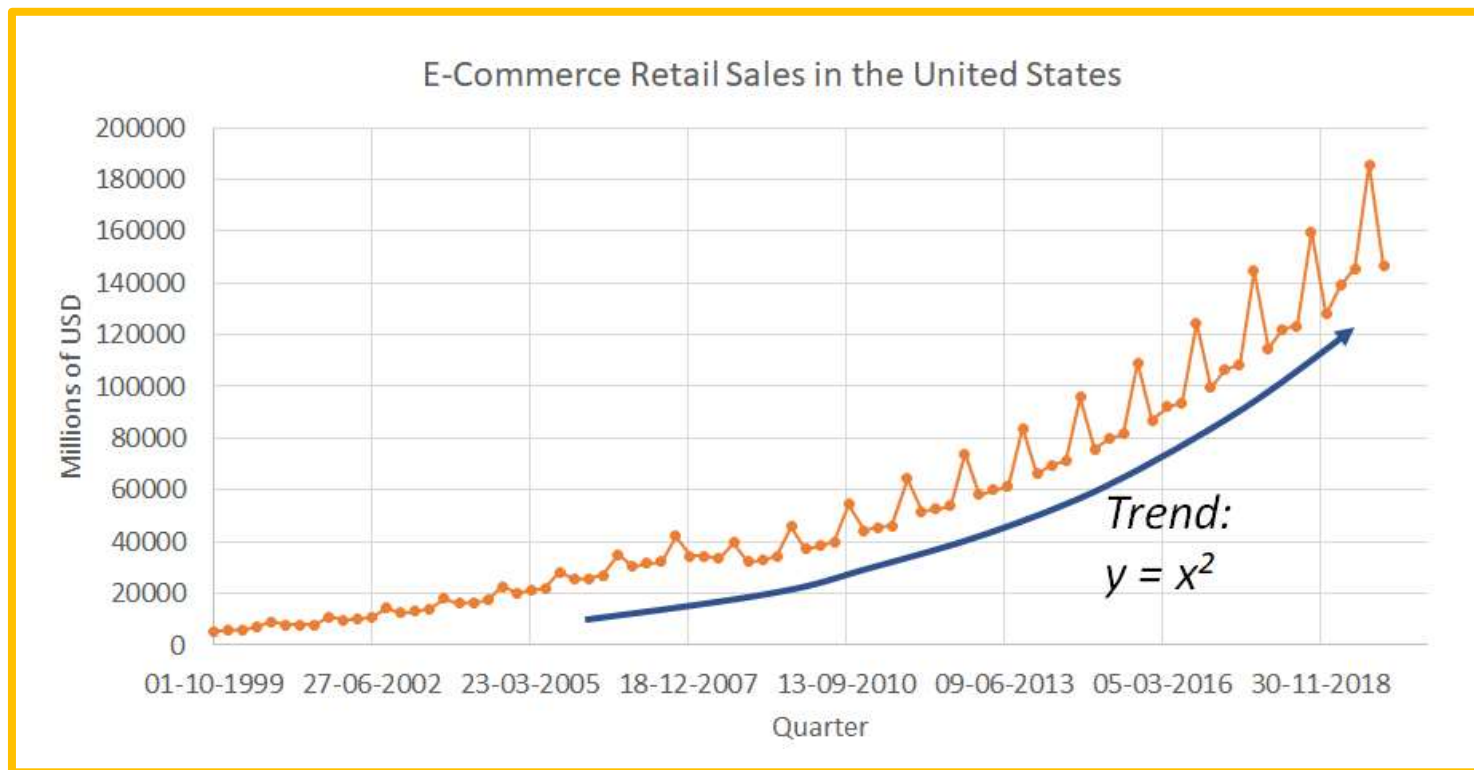
- 👉 level
- 👉 trend
- 👉 seasonality
- 👉 noise (non-systematic)



Time series patterns

- 1 **Trend** exists when there is a **long-term increase or decrease** in the data over a **period** that persists over a long time. The trend can be linear or non-linear.

The **trend** component



Time series patterns

① **Trend** exists when there is a long-term increase or decrease in the data. The trend can be linear or non-linear.

① Trend

There are **two types** of classification in the trend:

⊕ **Deterministic trend**

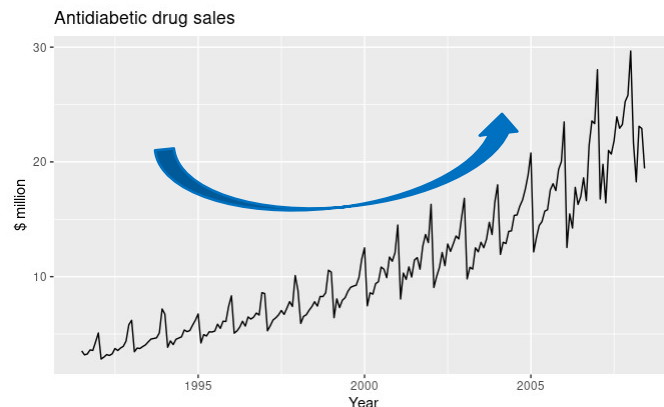
These are trends in which the value of the **time-dependent variable** increases or decreases **consistently**.

✧ **Non-deterministic trend (stochastic trend)**

These are trends in which the value of the time-dependent variable increases or decreases **inconsistently**.



Monthly sales of **antidiabetic drugs** in Australia



There is also a strong seasonal pattern that increases in size as the level of the series increases.

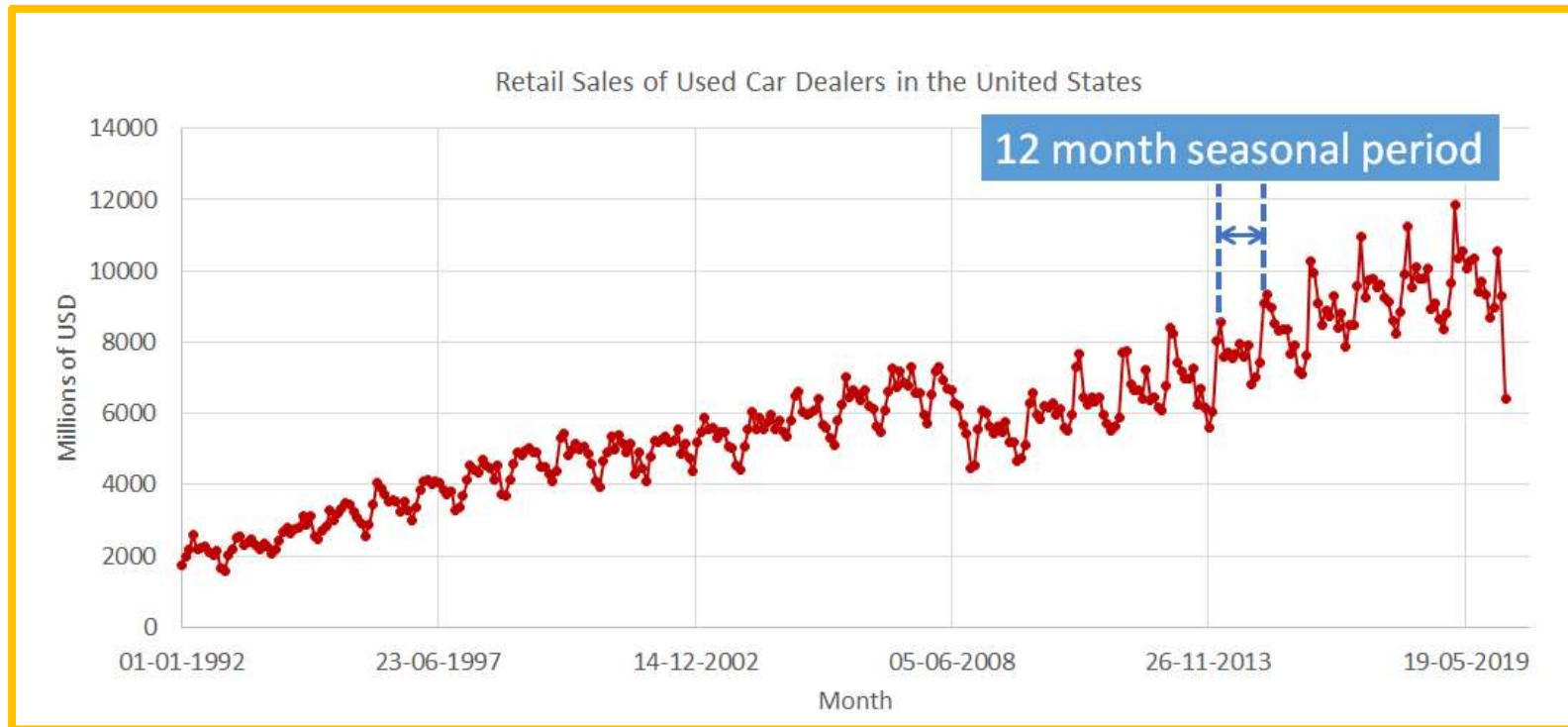
The sudden drop at the start of each year is caused by a **government subsidization scheme** that makes it cost-effective for patients to stockpile drugs at the end of the calendar year.

Time series patterns

② **Seasonality** (seasonal variations) patterns occur when a time series is affected by **seasonal factors** such as the time of the year or the day of the week.

Seasonality is always of a **fixed** and **known frequency**.

The seasonal component



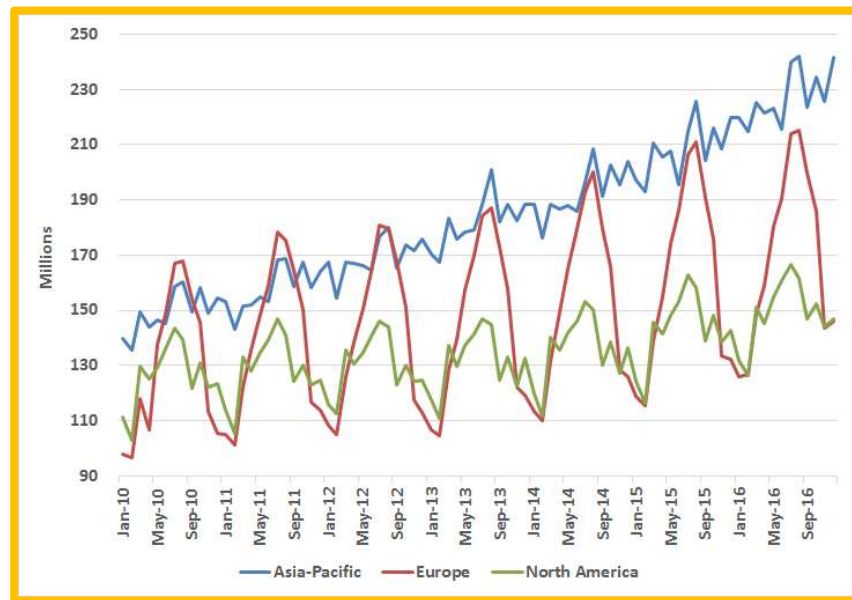
Time series patterns

Seasonality (Seasonal variations)

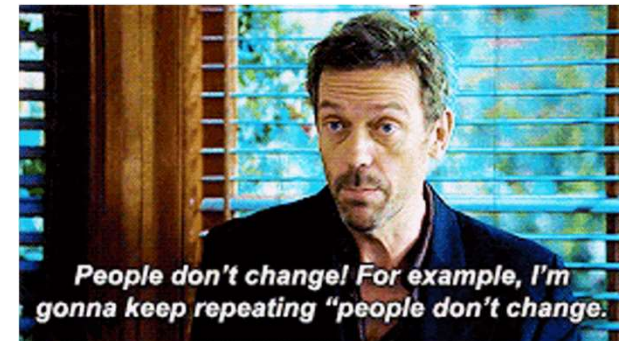
It is the regular pattern of up and down fluctuations in a time series. It may be a short-term variation occurring due to seasonal factors.

Recurring patterns, e.g., produced by **humans habits**.

Monthly passenger traffic by region (2010-2016)



② **Seasonality** (seasonal variations) patterns occur when a time series is affected by **seasonal factors** such as the **time of the year or the day of the week**. Seasonality is always of a **fixed and known frequency**.



The main sources of seasonality are

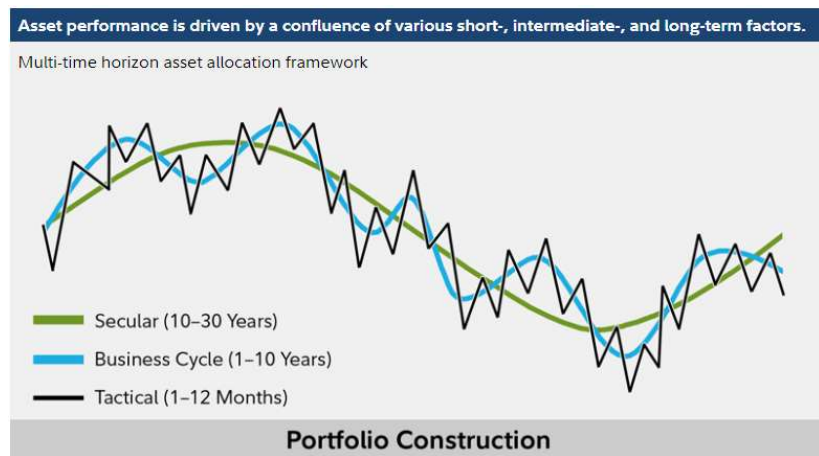
- ☞ Climate
- ☞ Institutions
- ☞ Social habits and practices
- ☞ Calendar

Time series patterns

③ Cyclicity (Cyclical fluctuations)

It can be defined as a medium-term variation caused by circumstances that **repeat in irregular intervals**. The cycle rises and falls without a fixed frequency. The duration of these fluctuations is usually of at least 2 years.

A cycle occurs when the data exhibit rises and falls that are not of a fixed frequency.



Examples

- ☞ The great recession from 2007-2008 (business cycle)
- ☞ The Spanish flu pandemic in 1918
- ☞ The COVID-19 crisis in 2020-2021

Many people confuse cyclic behavior with seasonal behavior, but they are really quite different. If the **fluctuations are not of a fixed frequency then they are cyclic**. If the frequency is unchanging and associated with some aspect of the calendar, then the pattern is seasonal. In general, the average length of cycles is longer than the length of a seasonal pattern, and the magnitudes of cycles tend to be more variable than the magnitudes of seasonal patterns.

Time series patterns

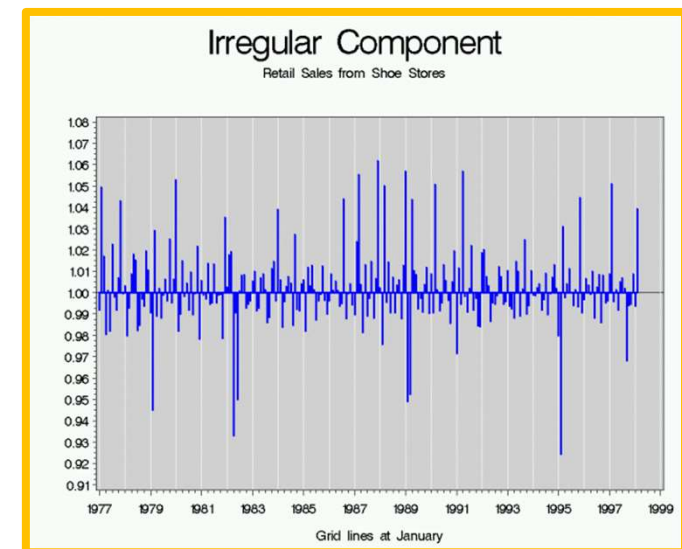
④ Irregularity (Irregular variations)

It refers to variation occurs due to unpredictable factors and also do not repeat in particular patterns.

Anything not included in the trend-cycle or the seasonal effects (or in estimated trading day or holiday effects). Its values are unpredictable in regards to timing, impact, and duration. It can arise from sampling error, non-sampling error, unseasonable weather, natural disasters, strikes, etc.

Examples

- ➡ Economical variations caused due to natural disasters.
- ➡ Irregular component from retail sales from shoe stores.



Time series patterns

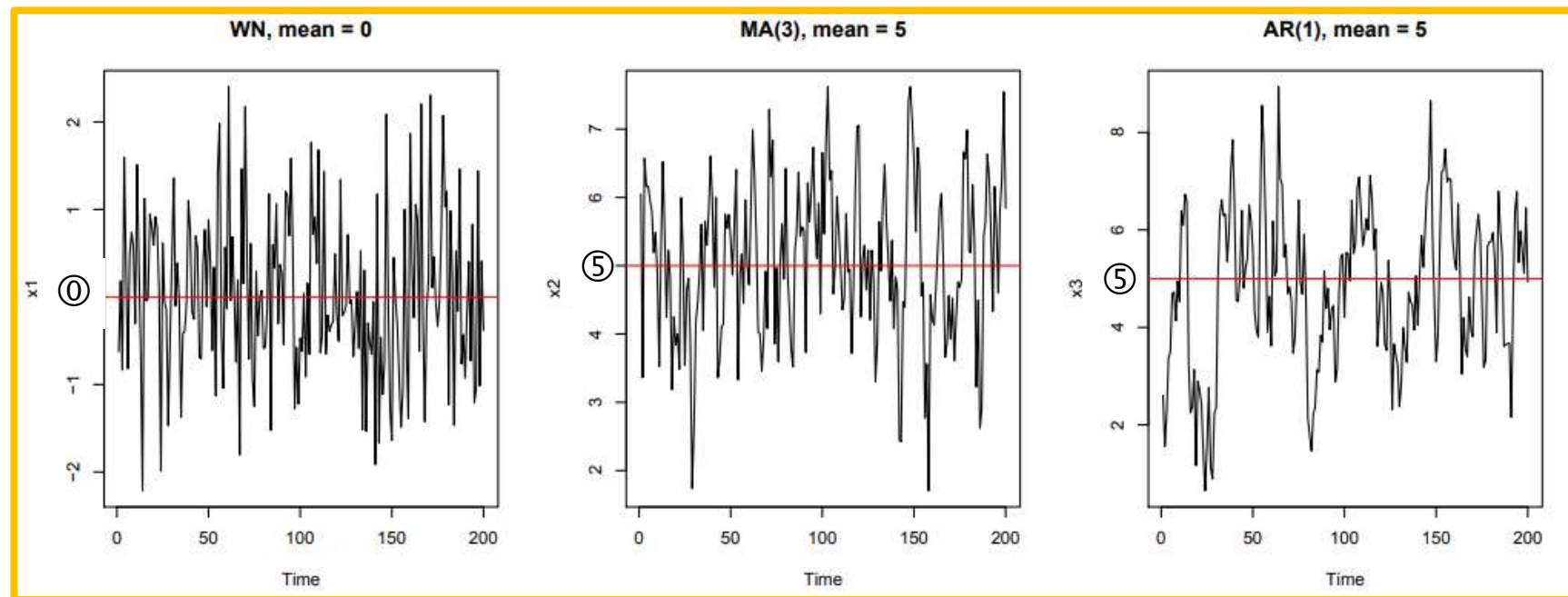
* Stationary time series and non-stationary time series



A **stationary series (process)**: a random (stochastic) process with a **constant** mean, variance and covariance.

Examples of **stationary** time series.

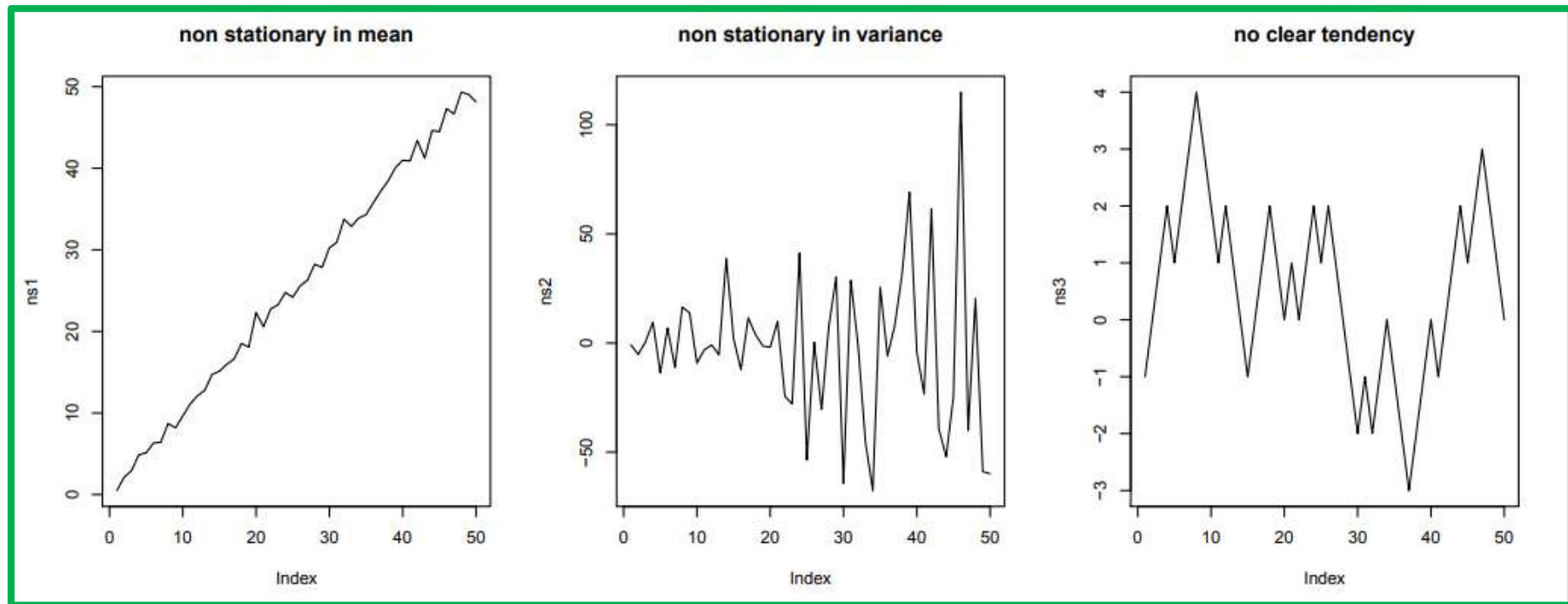
WN: White Noise
MA: Moving Average
AR: AutoRegressive



Time series patterns

* Stationary time series and non-stationary time series

Examples of **non-stationary** time series.

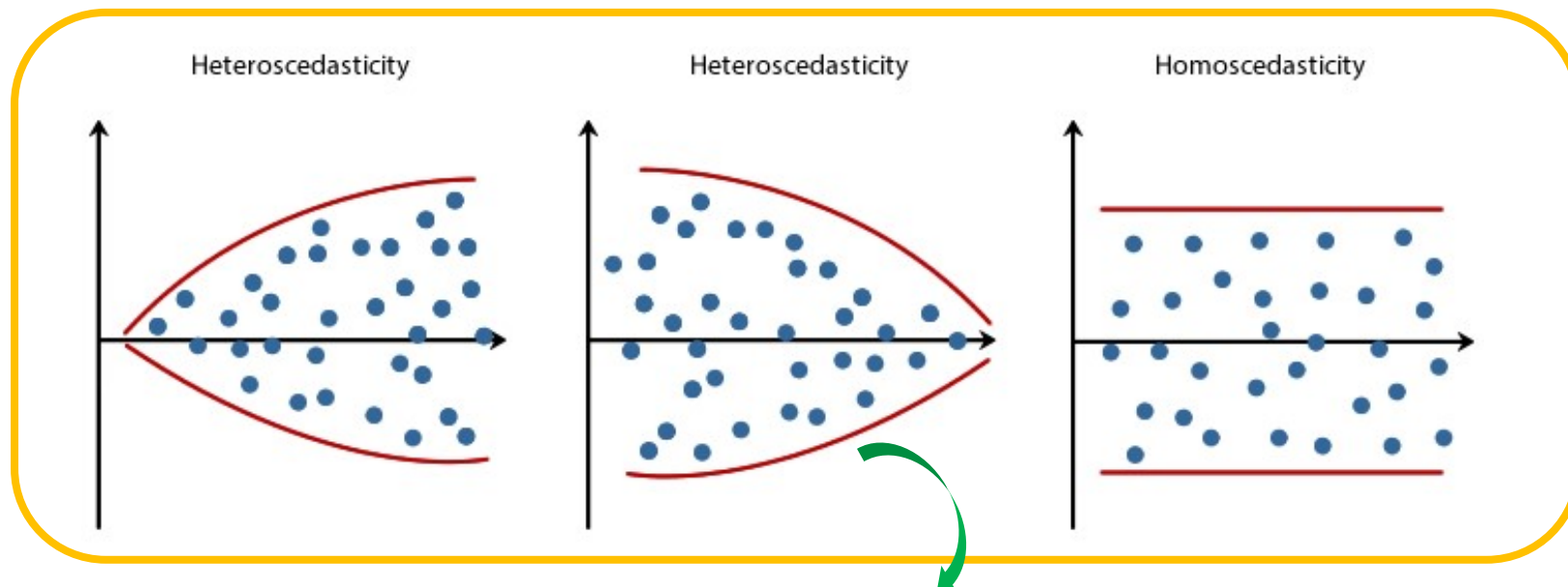


Time series patterns

Stationary time series and non-stationary time series

Heteroscedasticity is what you have in your data when the *conditional variance* in your data is **not** constant.

Conditional variance is the variability that you see in the dependent variable y for each value of the explanatory variables X , or each value of time period t (in case of time series data).



GARCH (Generalized AutoRegressive Conditional Heteroskedasticity) model

Tests for identifying stationarity in time series



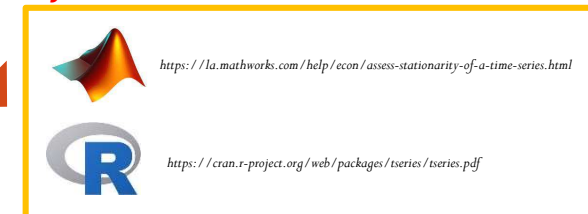
✓ **DF (Dickey-Fuller) test**

This is one of the statistical tests for **checking the presence of stationarity**.

Null hypothesis: Time series is non-stationary.

Alternative hypothesis: Time series is stationary.

```
from statsmodels.tsa.stattools import adfuller
```



✓ **ADF (Augmented Dickey-Fuller) Test**

It is used for checking the presence of a **unit root** in time series (AR model).

Null hypothesis: The series has a unit root which means series is non-stationary.

Alternative hypothesis: The series has no unit root which means series is stationary.

✓ **KPSS (Kwiatkowski-Phillips-Schmidt-Shin) Test**

It is another statistical test used for **checking the presence of stationarity**.

Null hypothesis: The series stationary.

Alternative hypothesis: The series has a unit root which means series is non-stationary.

It has to be noted that the null and alternative hypothesis is the opposite for ADF and KPSS test.

```
from statsmodels.tsa.stattools import kpss
```

Dickey, D.A. and W.A. Fuller (1979). Distribution of the estimators for autoregressive time series with a unit root, Journal of the American Statistical Association, vol. 74, no. 166, pp. 427-431. <http://www.jstor.org/pss/2286348>

Kwiatkowski, D., Phillips, P.C.B., Schmidt, P., & Shin, Y. (1992). Testing the null hypothesis of stationarity against the alternative of a unit root. Journal of Econometrics, 54: 159-178. <https://www.sciencedirect.com/science/article/abs/pii/030440769290104Y>

Time series patterns

* Complex patterns: Chaos theory

Chaos theory is related for a dynamical system whose apparently random states of disorder and irregularities are actually governed by underlying patterns and deterministic laws that are **highly sensitive to initial conditions**.

Chaos theory patterns:

- ☞ interconnectedness
- ☞ constant feedback loops
- ☞ repetition
- ☞ self-similarity
- ☞ fractals
- ☞ self-organization



Lord Robert May
(1936-2020)

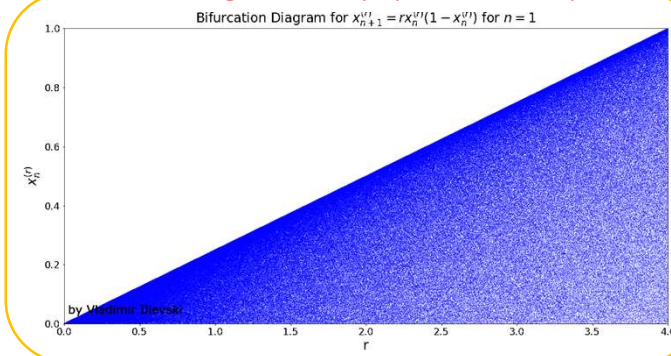


Aleksandr Mikhailovich Lyapunov
(1857-1918)

To characterize the attractor:

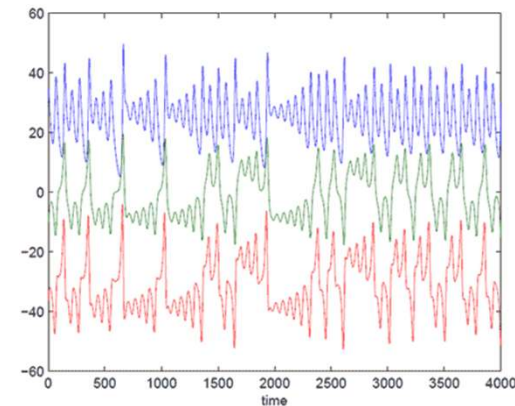
- ✓ Dimension quantifies the self-similarity of a geometrical object (Higuchi, Katz, Petrosian,...)
- ✓ A positive maximal **Lyapunov exponent** is a strong signature of chaos

Logistic map (1838, 1976)

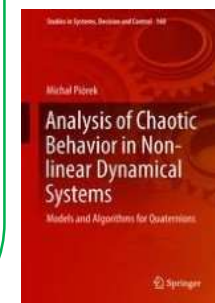
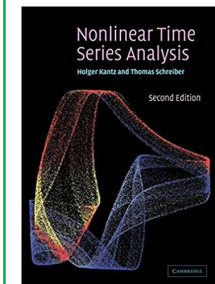
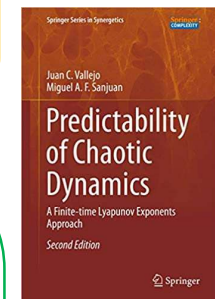
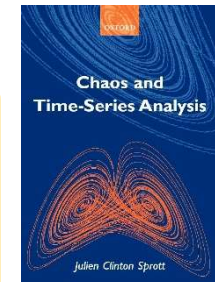
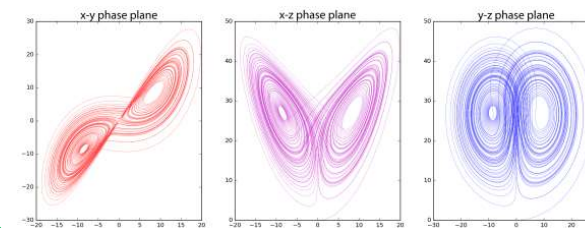


Lorenz attractor (1963)

$$\frac{dx}{dt} = \sigma(y - x) \quad \frac{dy}{dt} = x(\rho - z) - y \quad \frac{dz}{dt} = xy - \beta z$$



weather prediction



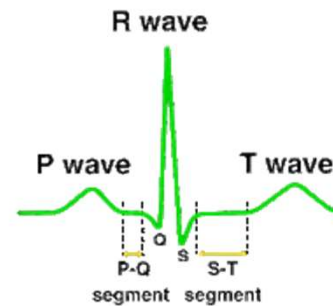
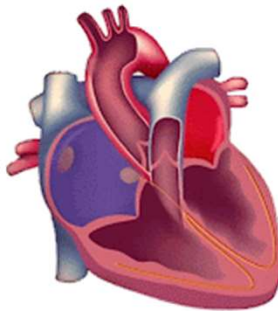
Time series patterns

* Complex patterns

Electrocardiogram (ECG)



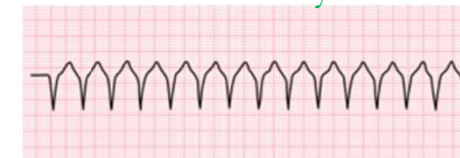
anomaly detection



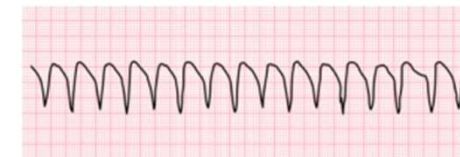
ventricular fibrillation



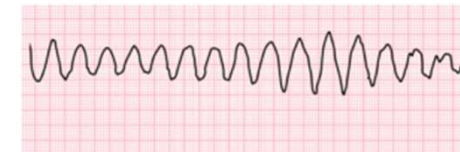
ventricular tachycardia



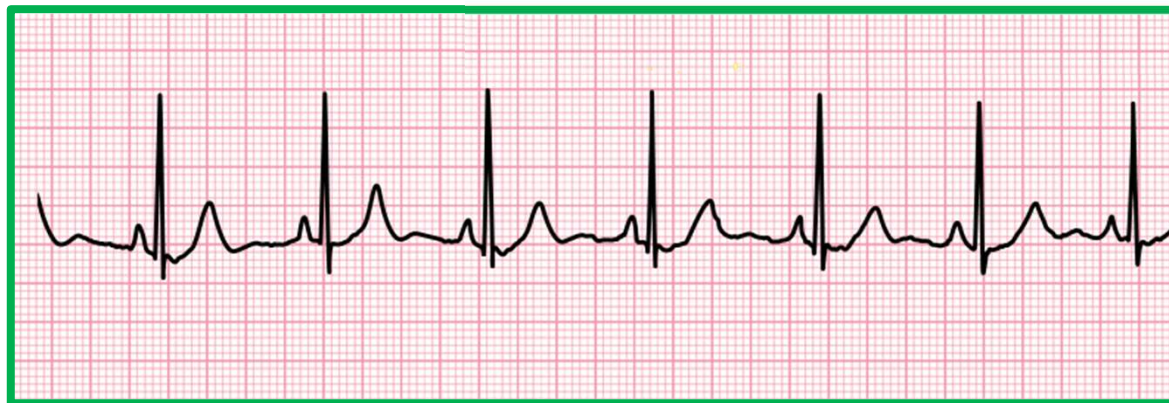
ventricular tachycardia monomorphic



ventricular tachycardia polymorphic



ventricular tachycardia torsade de Pointes



Wolff-Parkinson-White syndrome

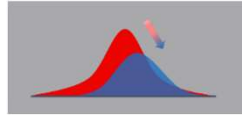


wandering atrial pacemaker



Time series patterns

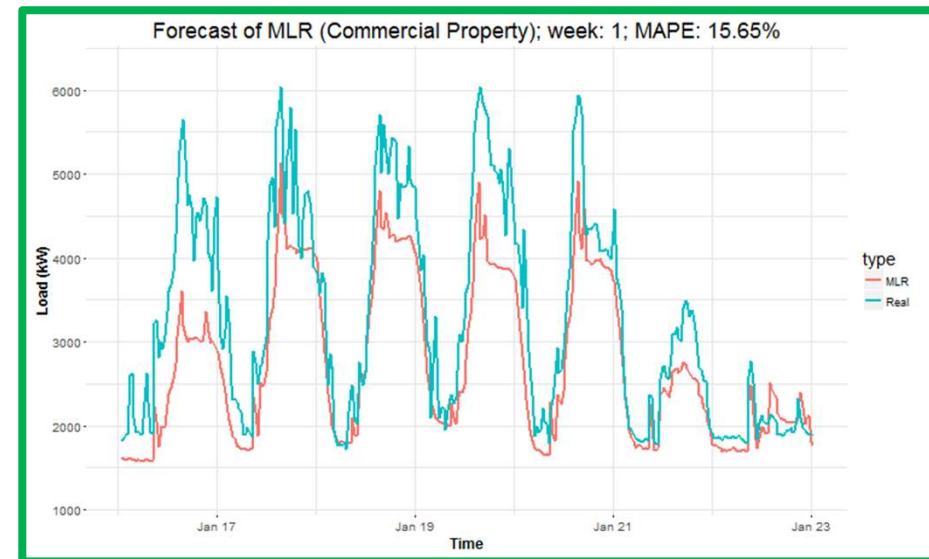
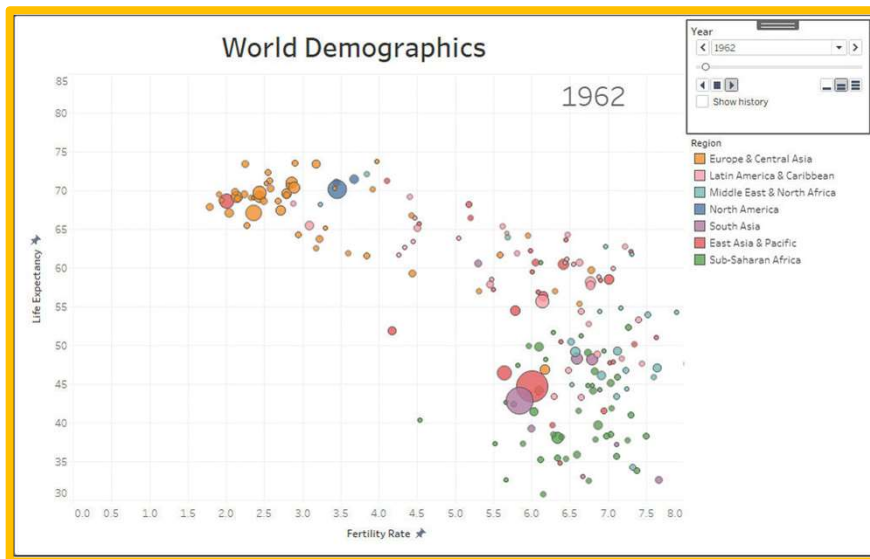
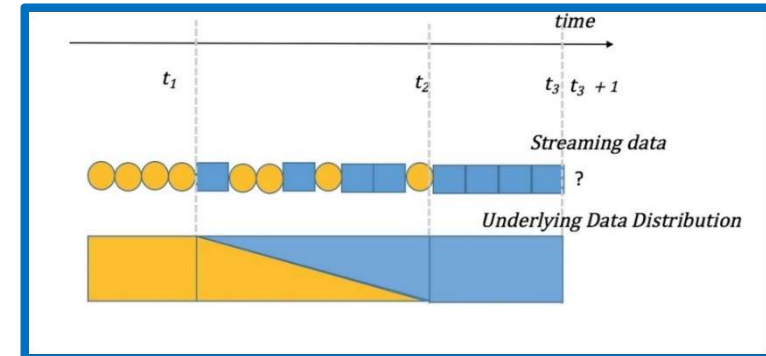
Concept drift

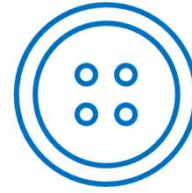


Heraclitus, the Greek philosopher said,
“**Change** is the only **constant** in life.”

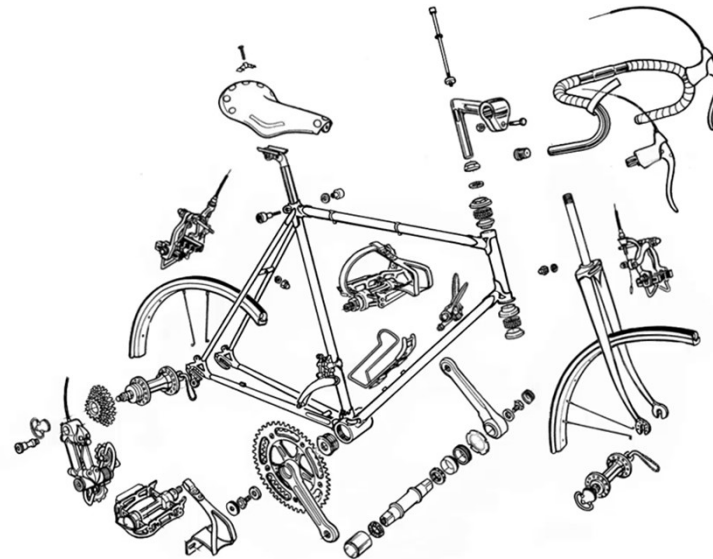
Data can **change** over time. This can result in **poor and degrading predictive performance** in predictive models that **assume a static relationship** between input and output variables.

The term **concept** refers to the quantity to be predicted.



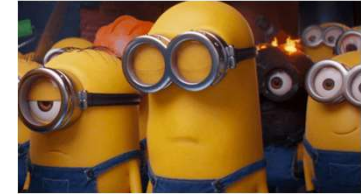


Time series decomposition



Time series decomposition involves thinking of a series as a **combination** of level, trend, seasonality, and noise **components**.

Some terminologies



Decomposition

A proper time series analysis demands the data to be in a stationary way but we rarely get data with stationary characteristics. Hence, the non-stationary time series converted to a stationary series by removing its trend and seasonality. This process of **removing trend and seasonality** is called **Time series decomposition**.



Differencing

Differencing is a decomposition process through which **trend and seasonality are eliminated**. Here, we usually take the difference of observation with particular instant with previous instant.

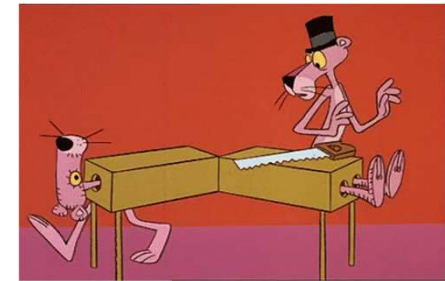


Transformation

Level: The average value in the series.

Transformations can help to **stabilize the variance** of a time series. Transformation is the easiest way to remove trends from a time series by converting the data into different scales using various operations like **logarithms**, **square roots**, etc. However, it is rarely implemented over differencing due to the possibility of a loss of information.

Pattern decomposition



There are **two main methods** among other methods to decompose seasonality:

- ✓ linear models with additive or multiplicative models, and
- ✓ season-trend decomposition using **LOESS**

Local regression: LOESS (locally estimated scatterplot smoothing)

The `seasonal_decompose` model uses moving averages to decompose seasonality trends.

The **additive** models has following format:

$$\text{Time Series} = \text{Level} + \text{Trend} + \text{Seasonality} + \text{Residual}$$

linear

The **multiplicative** time series model has the following format:

$$\text{Time Series} = \text{Level} * \text{Trend} * \text{Seasonality} * \text{Residual}$$

nonlinear

LOESS

LOcally Estimated Scatterplot Smoothing

LOESS (1979) which is commonly referred to as **Savitzky-Golay smoothing filter (1964)**.

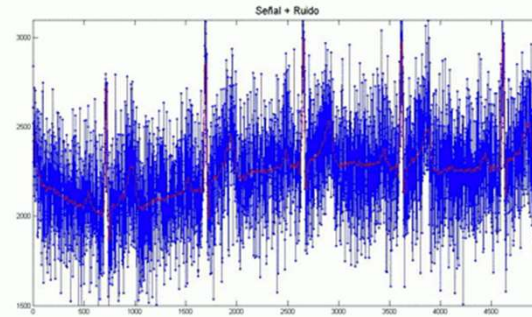
A Savitzky-Golay filter is a **digital filter** that can be applied to a set of digital data points for the purpose of **smoothing** the data, that is, to increase the precision of the data without distorting the signal tendency. This is achieved, in a process known as convolution, by fitting successive sub-sets of adjacent data points with a low-degree polynomial by the method of linear least squares.

- 1 Select a **window** (say, five points) around that point
- 2 Fit a **polynomial** to the points in the selected window
- 3 Replace the data point in question with the corresponding value of the **fitted polynomial**.

William S. Cleveland **rediscovered** the method in **1979** and gave it a **distinct name**: **LOWESS** (LOcally **W**eighted Scatterplot Smoothing)

Savitzky, A., Golay, M.J.E. (1964). Smoothing and differentiation of data by simplified least squares procedures, Analytical Chemistry. 36(8): 1627-1639. <https://pubs.acs.org/doi/10.1021/ac60214a047>

Cleveland, W. S. (1979). Robust locally weighted regression and smoothing scatterplots, Journal of the American Statistical Association, 74(368): 829-836. <https://www.tandfonline.com/doi/abs/10.1080/01621459.1979.10481038>



Abraham Savitzky
(1919-1999)



Marcel Jules Edouard Golay
(1902-1989)



William Swain Cleveland
(1943 -)

2001: Data
science lecture

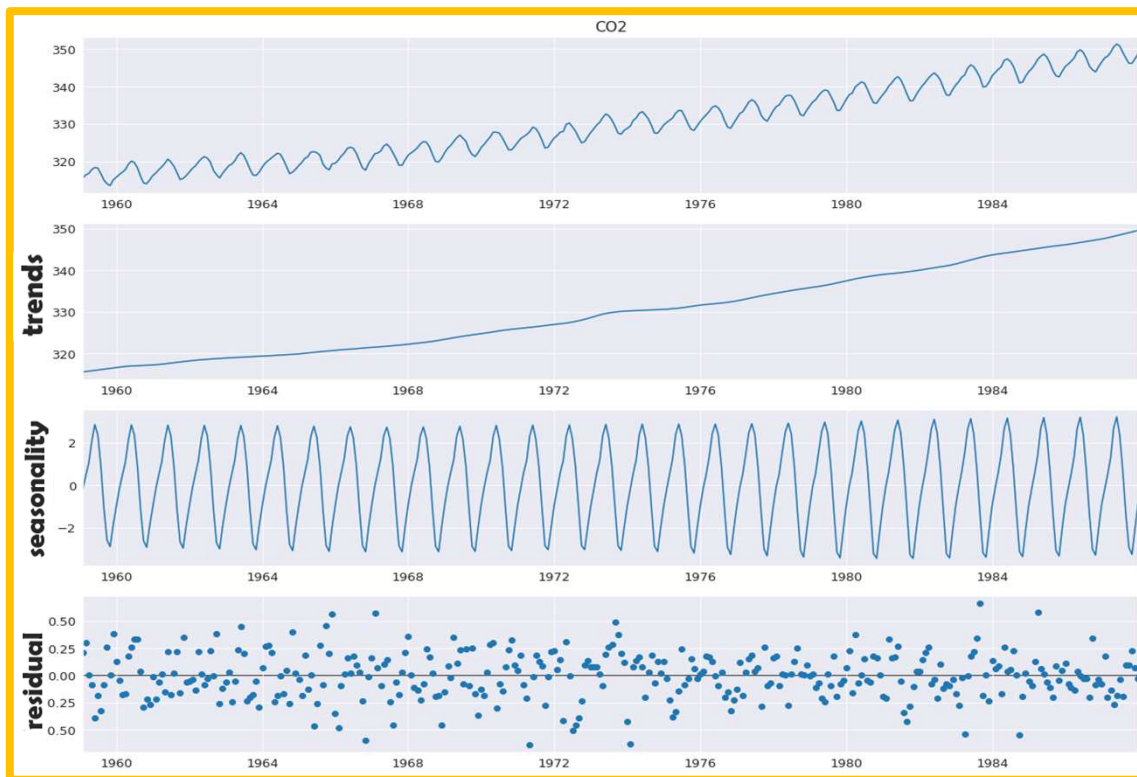


Some important terminologies



Decomposition

STL (Seasonal and Trend decomposition using Loess)



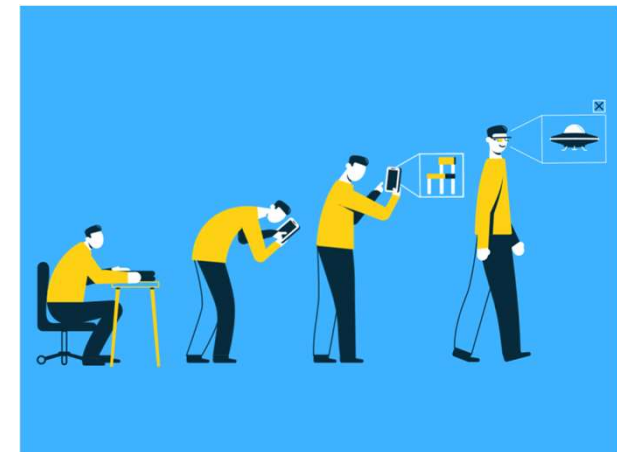
Cleveland et al. (1990) uses CO₂ data. This monthly data (January 1959 to December 1987) has a clear trend and seasonality across the sample.

Cleveland, R. B., Cleveland, W. S., McRae, J. E., & Terpenning, I. J. (1990). STL: A seasonal-trend decomposition procedure based on loess. *Journal of Official Statistics*, 6(1), pp. 3-33. <http://bit.ly/stl1990>

$y = \text{trends} + \text{seasonality} + \text{remainder (residual, error)}$

$$y_t = f(S_t, T_t, R_t)$$

where y_t = data at period t
 T_t = trend-cycle component at period t
 S_t = seasonal component at period t
 R_t = remainder component at period t

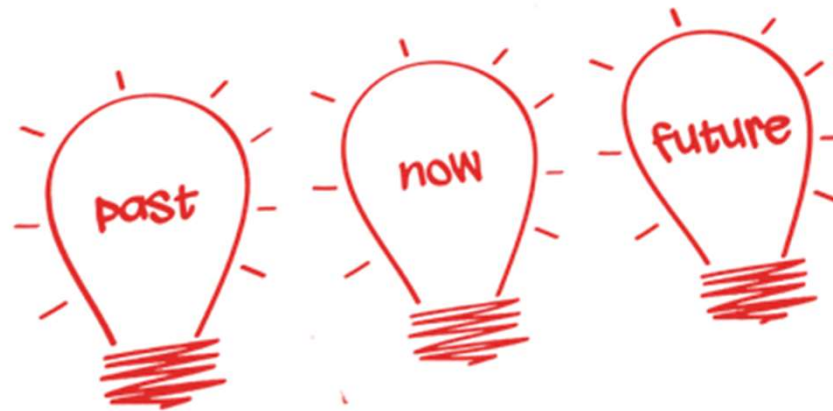


Time series must

- Be seasonal
- Have at least two full periods



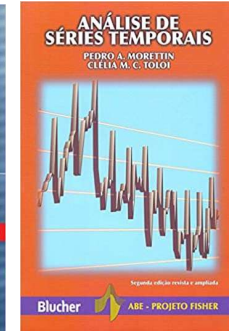
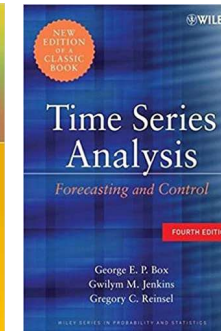
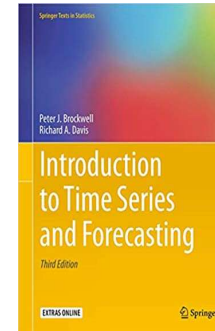
Time series forecasting



Classical (statistical) models

✓ Simple exponential smoothing

$$y(t) = \alpha \cdot x(t) + (1 - \alpha) \cdot y(t-1)$$



✓ Double exponential smoothing or **Holt** linear method

$$y(t) = \alpha \cdot x(t) + (1 - \alpha) \cdot \{y(t-1) + b(t-1)\}$$
$$b(t) = \beta \cdot \{y(t) - y(t-1)\} + (1 - \beta) \cdot b(t-1)$$

✓ Triple exponential smoothing or **Holt-Winters** method

$$y(t+h) = l(t) + h \cdot b(t) + s(t+h-L[k+1])$$
$$l(t) = \alpha \cdot \{x(t) - s(t-L)\} + (1 - \alpha) \cdot \{l(t-1) + b(t-1)\}$$
$$b(t) = \beta \cdot \{l(t) - l(t-1)\} + (1 - \beta) \cdot b(t-1)$$
$$s(t) = \gamma \cdot \{x(t) - l(t-1) - b(t-1)\} + (1 - \gamma) \cdot \{s(t-L)\}$$

additive
seasonality
 α, β, γ tuning

Additive time series model

$$y(t) = T(t) + S(t) + R(t)$$

Multiplicative time series model

$$y(t) = T(t) \cdot S(t) \cdot R(t)$$

With the help of logarithms it is possible to pass from the **multiplicative model** to the **additive model**.

Brown, R. G. (1959). Statistical forecasting for inventory control. McGraw/Hill, NY, USA.

Holt, C. E. (1957). Forecasting seasonals and trends by exponentially weighted averages (O.N.R. Memorandum No. 52).

Carnegie Institute of Technology, Pittsburgh USA. <https://www.sciencedirect.com/science/article/abs/pii/S0169207003001134?via%3Dihub>

Winters, P. R. (1960). Forecasting sales by exponentially weighted moving averages. Management Science, 6(3), 324–342.

<https://pubsonline.informs.org/doi/abs/10.1287/mnsc.6.3.324>

- ① Overall smoothing
- ② Trend smoothing
- ③ Seasonality smoothing



Time series forecasting using **linear** models

Forecasting time series using **linear** models with **eXogenous inputs (X)**

1952 Z-Transform (correlation of $x[n]$ with z^n):

$$X(z) = \sum_{n=0}^{\infty} x[n]z^{-n}$$

Example: **ARMAX** models

AR

X

MA

$$\mathbf{A}(z).y(t) = \mathbf{B}(z).u(t-nk) + \mathbf{C}(z).e(t)$$

z (or q)

Example of order tuning and delay:

$na = 2$ **A** polynomial order

$nb = 2$ **B** polynomial order

$nc = 2$ **C** polynomial order

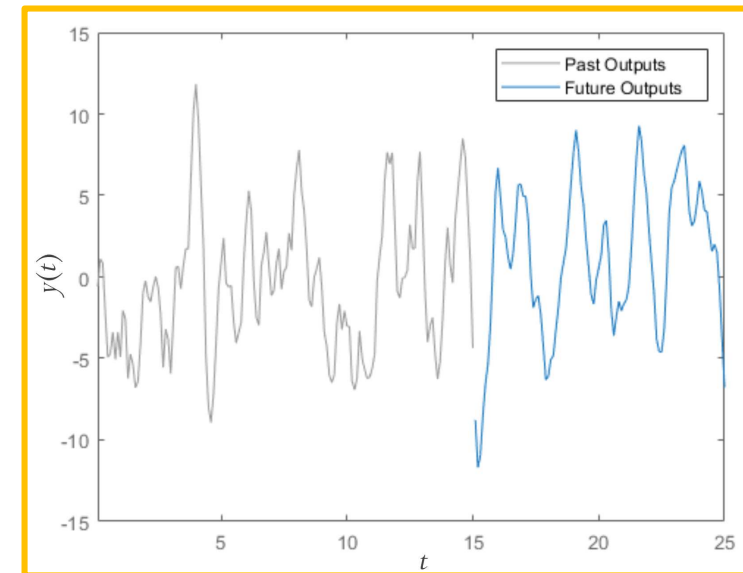
$nk = 1$ Input delay (transport delay, deadtime)

lags

$$\mathbf{A}(z) = 1 - 1.512 z^{-1} + 0.7006 z^{-2}$$

$$\mathbf{B}(z) = -0.2606 z^{-1} + 1.664 z^{-2}$$

$$\mathbf{C}(z) = 1 - 1.604 z^{-1} + 0.7504 z^{-2}$$



Ragazzini, J. R., Zadeh, L. A. (1952). The analysis of sampled-data systems. Transactions of the American Institute of Electrical Engineers, Part II: Applications and Industry. 71 (5): 225–234. <https://ieeexplore.ieee.org/document/6371274?signout=success>

Forecasting time series using linear models

AR

$$\hat{y}(t) = -a_1 y(t-1) - a_2 y(t-2)$$

MA

$$\hat{y}(t) = c_1 e(t-1) + c_2 e(t-2)$$

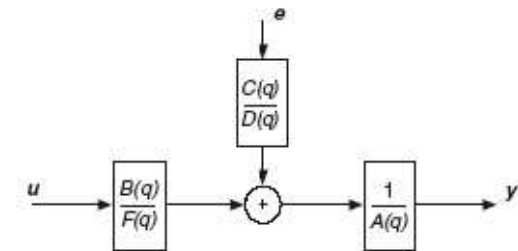
ARMA

$$\hat{y}(t) = -a_1 y(t-1) - a_2 y(t-2) + c_1 e(t-1) + c_2 e(t-2)$$

ARMAX

$$\hat{y}(t) = -a_1 y(t-1) - a_2 y(t-2) + c_1 e(t-1) + c_2 e(t-2) + b_1 u(t-1) + b_2 u(t-2)$$

$$\mathbf{y} = \mathbf{X}^T \hat{\boldsymbol{\theta}}$$



z or q

Data: 3 data and 2 unknowns

$$\mathbf{X} = \begin{bmatrix} 2 & -0.8 \\ -1.2 & 2 \\ -1 & 0.85 \end{bmatrix}, \mathbf{y} = \begin{bmatrix} 1.1 \\ 0.95 \\ -0.2 \end{bmatrix}$$

Find Least Squares solution to:

$$\begin{bmatrix} 2 & -0.8 \\ -1.2 & 2 \\ -1 & 0.85 \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} 1.1 \\ 0.95 \\ -0.2 \end{bmatrix}$$

Form variance/covariance matrix and cross correlation vector

$$\mathbf{X}^T \mathbf{X} = \begin{bmatrix} 6.44 & -4.85 \\ -4.85 & 5.36 \end{bmatrix}$$

$$\mathbf{X}^T \mathbf{y} = \begin{bmatrix} 1.26 \\ 0.85 \end{bmatrix}$$

Invert variance/covariance matrix

$$(\mathbf{X}^T \mathbf{X})^{-1} = \begin{bmatrix} 0.4870 & 0.4404 \\ 0.4404 & 0.5848 \end{bmatrix}$$

Least squares solution

$$\hat{\boldsymbol{\theta}} = [0.988 \quad 1.052]^T$$

$$\hat{\boldsymbol{\theta}} = (\mathbf{X}^T \mathbf{X})^{-1} \mathbf{X}^T \mathbf{y}$$

Others

ARI

The ARI (AutoRegressive Integrated) model is an AR model with an **integrator** in the noise channel.

$$A(q)y(t) = \frac{1}{1 - q^{-1}} e(t)$$

ARIX

(AutoRegressive Integrated with Extra Input)

Estimate the parameters of an ARIX model. An ARIX model is an ARX model with integrated noise.

$$A(q)y(t) = B(q)u(t - nk) + \frac{1}{1 - q^{-1}} e(t)$$

Forecasting time series using linear models

ARIMA models / Box-Jenkins

(model to **non-stationary** time series)

$$\Delta^D y_t = \sum_{i=1}^p \phi_i \Delta^D y_{t-i} + \sum_{j=1}^q \theta_j \epsilon_{t-j} + \sum_{m=1}^M \beta_m X_{m,t} + \epsilon_t$$
$$\epsilon_t \sim N(0, \sigma^2)$$

🔑 **ACF** How to detect autocorrelation?

Durbin-Watson test

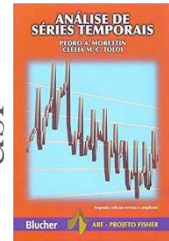
Bruesch-Godfrey test

🔑 **ACF and PACF**

De-trending the data

Identifying the significant terms

USP



PUC-Rio



p: The number of lag observations included in the model, also called the lag order (**AR order**).

d: The number of times that the raw observations are differenced, also called the degree of differencing.

q: The size of the moving average window, also called the order of moving average (**MA order**).

ACF: Auto Correlation Function

PCF: Partial Autocorrelation Function

✓ Identification of an **AR** model is often best done with the **PACF**.

✓ Identification of an **MA** model is often best done with the **ACF** rather than the PACF.

Box-Jenkins method

A. Model form selection

1. Evaluate stationarity
2. Selection of the differencing level (d) – to fix stationarity problems
3. Selection of the AR level (p)
4. Selection of the MA level (q)

B. Parameter estimation

C. Model checking

Variants

- 1 **Vector Auto Regression (VAR)**
- 2 **Vector Moving Average (VMA)**
- 3 **Vector Autoregressive Moving Average (VARMA)**
- 4 **Seasonal ARIMA (SARIMA)**
- 5 **SARIMA with exogenous inputs (SARIMAX)**

Time series models



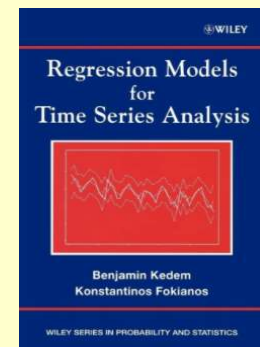
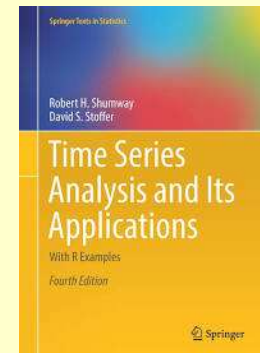
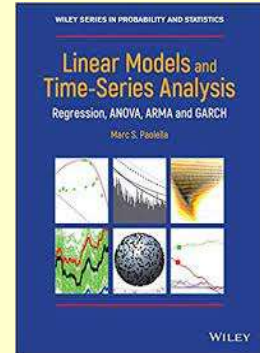
stationary models

- ARMAX: AutoRegressive Moving Average model with Exogenous inputs
- ARMA: AutoRegressive Moving Average model
- ARX: AutoRegressive model with Exogenous inputs
- AR: AutoRegressive model
- MA: Moving Average model

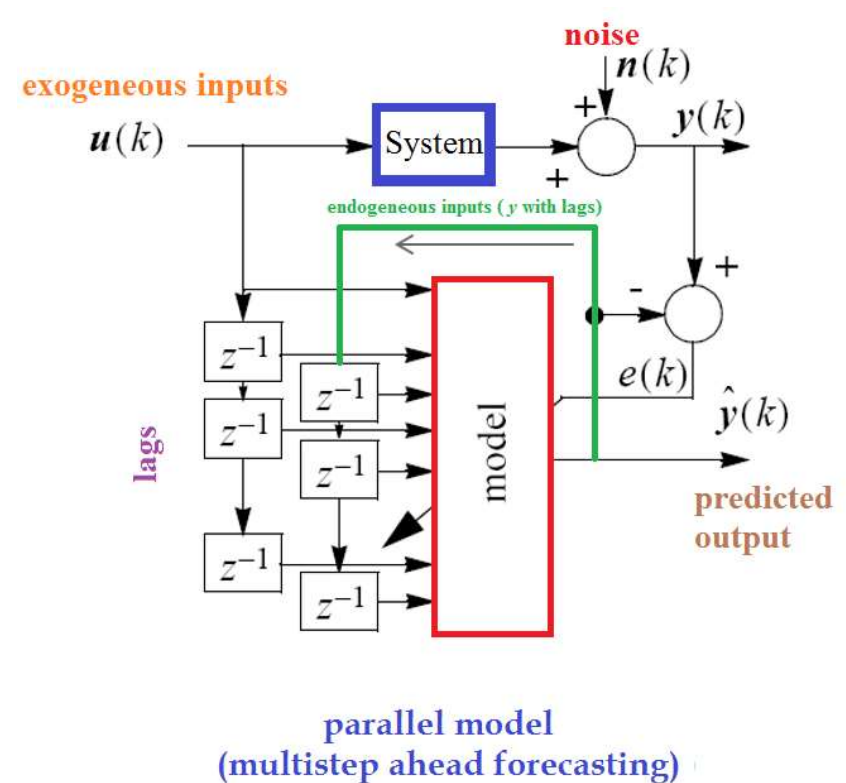
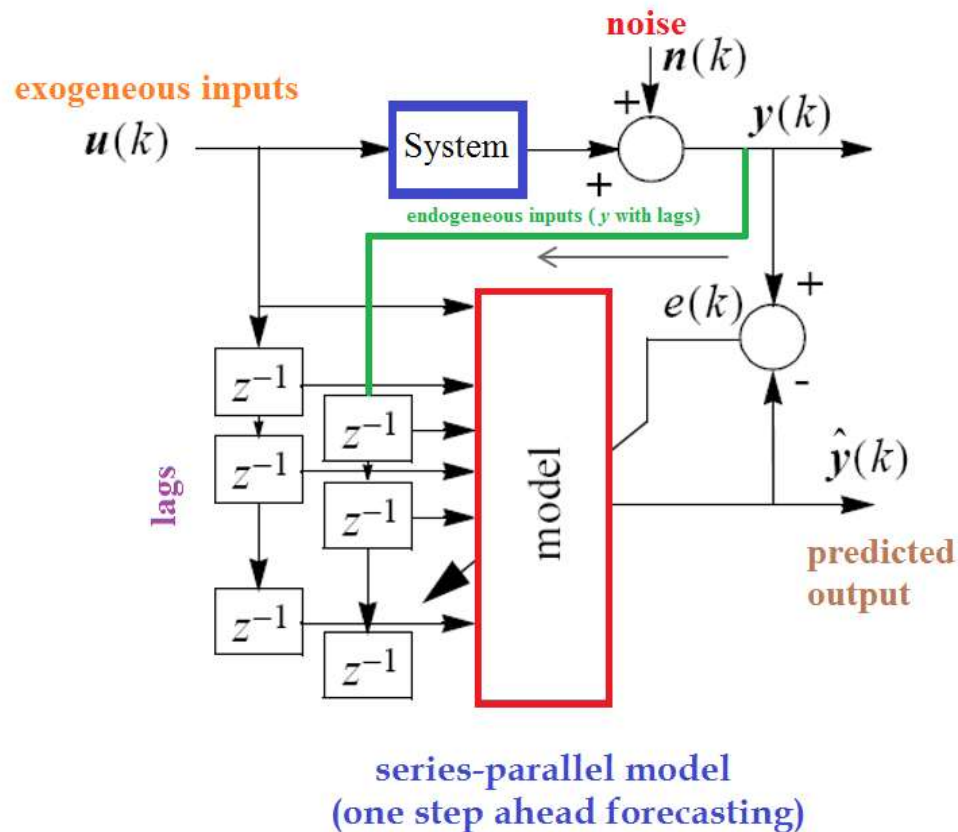


non-stationary models

- ARIMA: AutoRegressive Integrated Moving Average model
- ARCH, GARCH AutoRegressive Conditional Heteroskedasticity
 Generalized AutoRegressive Conditional Heteroskedasticity



Series-parallel and parallel models



J.V. Gorp (2000). Nonlinear identification with neural networks and fuzzy logic, PhD thesis, Vrije Universiteit Brussel, Dept. ELEC, 2000. [page 62](#)

Identification and Control of Dynamical Systems
Using Neural Networks

KUMPATI S. NARENDRA FELLOW, IEEE, AND KANNAN PARTHASARATHY

Why cross-validation different with time series?

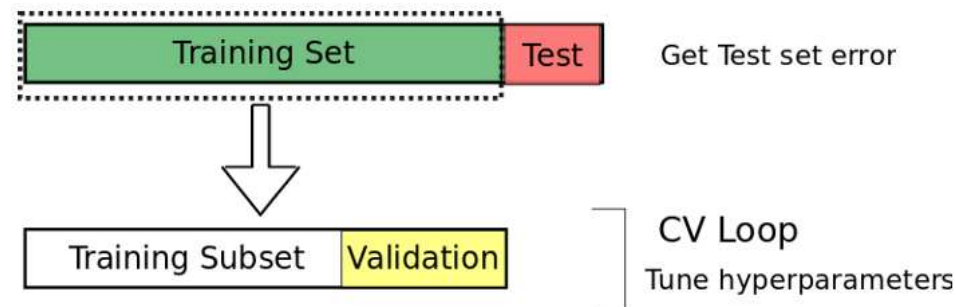
When dealing with time series data, traditional cross-validation (like k -fold) should not be used for two reasons:

1. Temporal dependencies

With time series data, particular care must be taken in splitting the data in order to prevent data leakage. In order to accurately simulate the “real world forecasting environment, in which we stand in the present and forecast the future” (Tashman 2000), the forecaster must withhold all data about events that occur chronologically after the events used for fitting the model.

So, rather than use k -fold cross-validation, for time series data we utilize **hold-out cross-validation** where a subset of the data (*split temporally*) is reserved for validating the model performance. For example, see **Figure 1** where the test set data comes chronologically after the training set. Similarly, the validation set comes chronologically after the training subset.

Figure 1



Tashman, L. J. Out-of-sample tests of forecasting accuracy: an analysis and review. *International Journal of Forecasting*, 16(4):437–450, 2000.
<https://www.sciencedirect.com/science/article/abs/pii/S0169207000000650>

Why cross-validation different with time series?

2. Arbitrary choice of test set

1 / 2

You may notice that the choice of the test set in **Figure 1** (Temporal dependencies) is fairly arbitrary, and that choice may mean that our test set error is a poor estimate of error on an independent test set.

To address this, we use a method called **Nested Cross-Validation**. Nested CV contains an outer loop for error estimation and an inner loop for parameter tuning (see **Figure 2 in the next slide**).

Advantageous:

*A nested cross-validation procedure provides an almost **unbiased** estimate of the true error. (Varma and Simon 2006)*

Varma, S., Simon, R. Bias in error estimation when using cross-validation for model selection. BMC Bioinformatics, 7(1):91, Feb 2006.
<https://bmcbioinformatics.biomedcentral.com/articles/10.1186/1471-2105-7-91>

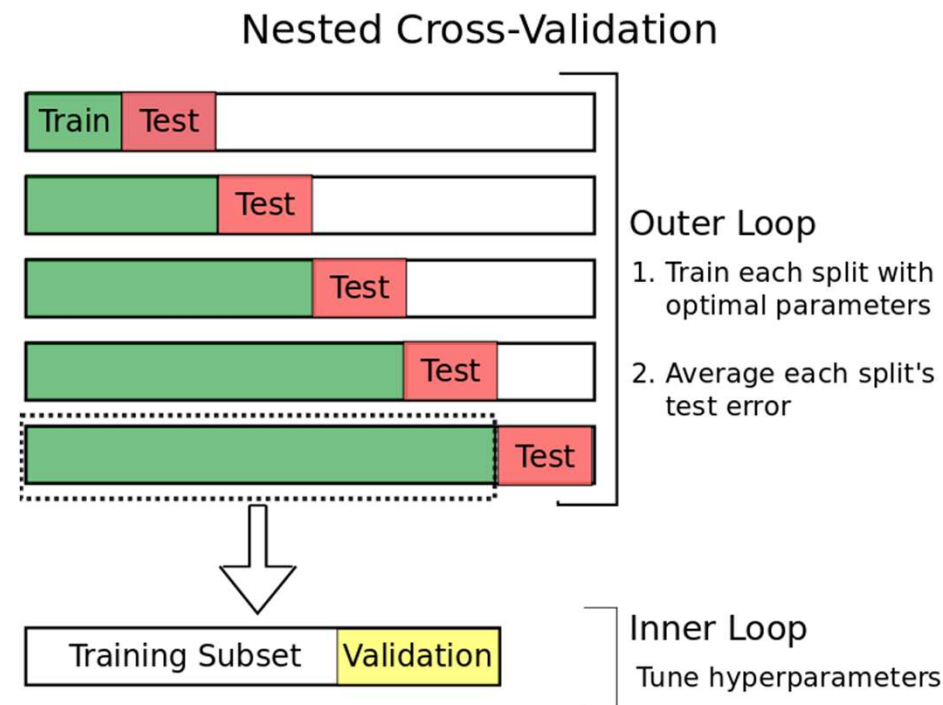
Why cross-validation different with time series?

2. Arbitrary choice of test set

2 / 2

The inner loop works exactly as discussed before: the training set is split into a training subset and a validation set, the model is trained on the training subset, and the parameters that minimize error on the validation set are chosen. However, now we add an outer loop which splits the dataset into multiple different training and test sets, and the error on each split is averaged in order to compute a robust estimate of model error.

Figure 2





Time series forecasting using **nonlinear** models

In general, a nonlinear black-box model can be considered as a concatenation mapping from previously observed data to a regressor space, which is followed by a **nonlinear function** expansion-type mapping to the space of the system's output.

Forecasting time series using **nonlinear** models

Forecasting Response of Nonlinear AR (or ARX or ARMAX) Models

A time series nonlinear AR (or ARX or ARMAX) model has the following structure:

$$y(t) = f(y(t-1), y(t-2), \dots, y(t-N)) + e(t)$$

where f is a **nonlinear function** with inputs $R(t)$, the model regressors.

The **regressors** can be the time-lagged variables $y(t-1), y(t-2), \dots, y(t-N)$ and their nonlinear expressions, such as $y(t-1)^2, y(t-1)y(t-2), \text{abs}(y(t-1))$.

Suppose that time series data from your system can be fit to a 2nd-order linear-in-regressor model with the following **polynomial regressors**:

$$R(t) = [y(t-1), y(t-2), y(t-1)^2, y(t-2)^2, y(t-1)y(t-2)]^T$$

Forecasting time series using **nonlinear** models

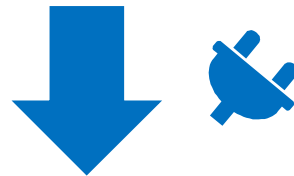
Forecasting Response of Nonlinear AR Models

The **nonlinear AR (or ARX or ARMAX)** model has the form:



$$y(t) = f(y(t-1), y(t-2), \dots, y(t-N)) + e(t)$$

$$R(t) = [y(t-1), y(t-2), y(t-1)^2, y(t-2)^2, y(t-1)y(t-2)]^T$$



$$y(t) = w_1 y(t-1) + w_2 y(t-2) + w_3 y(t-1)^2 + w_4 y(t-2)^2 + w_5 y(t-1)y(t-2) + c + e(t)$$

where the objective is estimate the model parameters (**weights**) W and **constant** (offset) c .

NARMAX model

NARMAX: Nonlinear Auto-Regression Moving Average model with eXogenous inputs

A popular structure selection approach used for identification of **NARX** (Nonlinear AutoRegressive eXogenous) **or** **NARMAX** (Nonlinear AutoRegressive, Moving Average eXogenous) models is the class of **Orthogonal Least Squares (OLS)** type algorithms, introduced in **Billings et al. (1988)**, which can be considered as wrapper method.

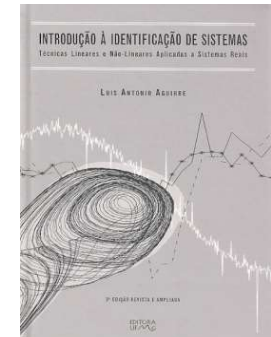
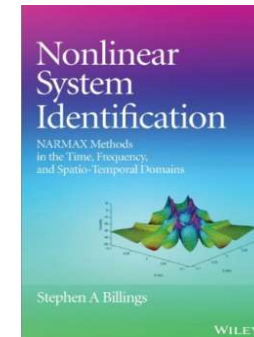
This approach enables the identification of the model structure for Linear In-the-Parameter (LIP) models by evaluating the relevance of individual model terms in an orthogonal space in a forward regression manner. For the measure of quality, the error reduction ratio is used (see, e.g., **Billings, 2013**).



Stephen A. Billings
Department of Automatic
Control and Systems Engineering
University of Sheffield



Luis A. Aguirre
Engenharia Elétrica
UFMG



Billings, S.A. (2013). Nonlinear system identification – NARMAX methods in time, frequency, and spatiotemporal domains. Wiley.

Billings, S.A., Korenberg, M.J., Chen, S. (1988). Identification of non-linear output affine system using an orthogonal least-squares algorithm. Int. J. of Systems Science, 19(8), 1559-1568.

Korenberg, M., Billings, S.A., Liu, Y.P., McIlroy, P.J. (1988). Orthogonal parameter estimation algorithm for non-linear stochastic systems. Int. J. Control 48, 193-210.

NARMAX model

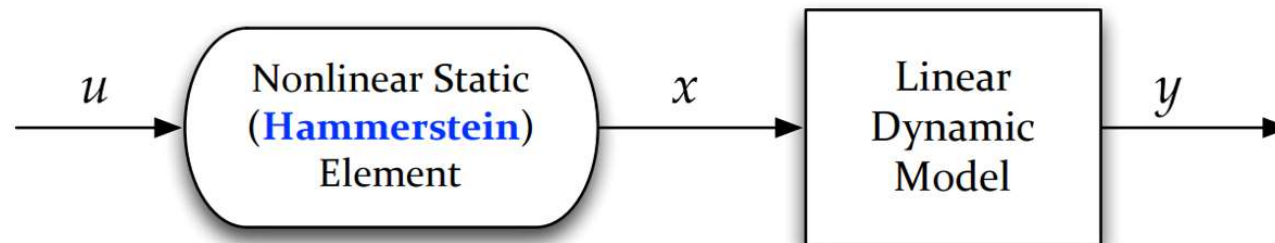
The initial structure of the NARMAX polynomial is determined by d , N_u , N_y , N_e , and l .

A clear disadvantage of polynomial models is the enormous number of terms a general nonlinear polynomial may have.

The significance of a model term is expressed using its corresponding **Error Reduction Ratio (ERR)**. This value is calculated for every term as part of the NARMAX estimation process.

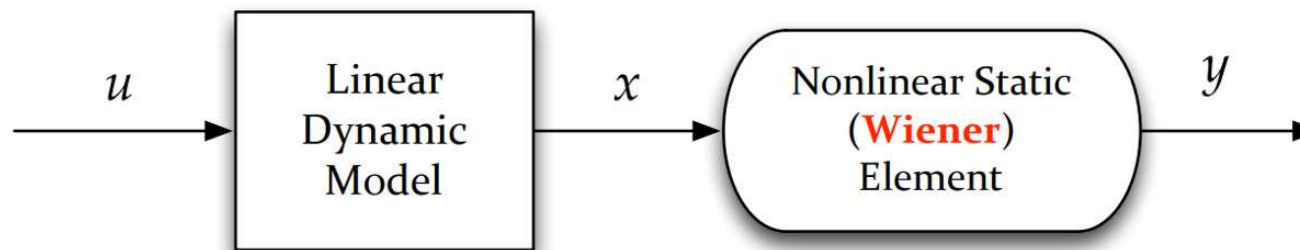
The ERR is an indication of the reduction in the model's prediction error that occurs when the model term considered is introduced into the model. This reduction is expressed in proportion to the maximum error (a constant) that results from removing all the terms from the model. The value of the ERR is therefore proportional to the significance of the term to which it corresponds.

Block-oriented models



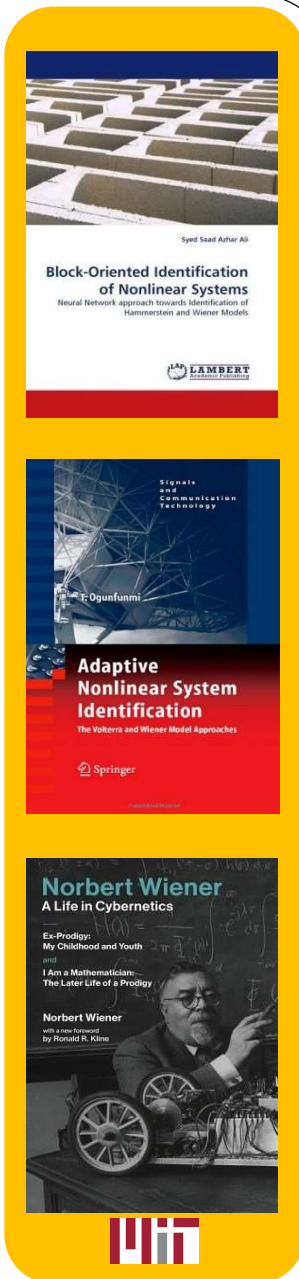
Hammerstein model

K.S. Narendra, P.G. Gallman An iterative method for the identification of nonlinear systems using a Hammerstein model, IEEE Trans. Aut. Control, Vol. AC-11, pp. 546-550, July 1966. <https://www.sciencedirect.com/science/article/pii/S0005109817303990#fig2a?>



Wiener model

M. Shoukens, K. Tiels Identification of block-oriented nonlinear systems starting from linear approximations: A survey, Automatica, Vol. 85, pp. 272-292, 2017. <https://www.sciencedirect.com/science/article/pii/S0005109817303990#fig2a?>

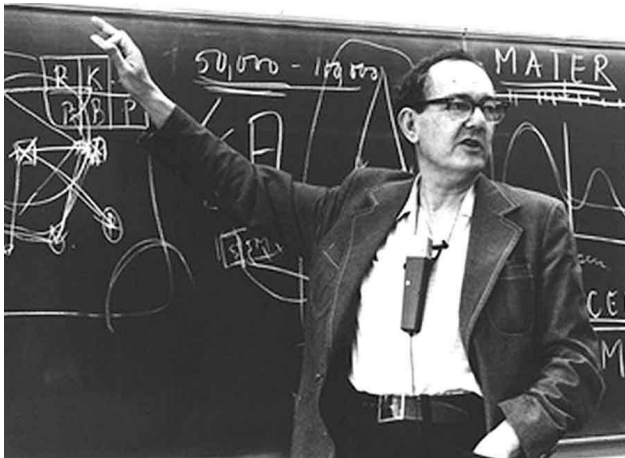


Time series forecasting using machine learning

Machine learning

Learning is any process by which a system improves performance from experience.

Machine Learning is concerned with **computer programs that automatically** improve their performance through **experience**.



Herbert Alexander Simon
(1916-2001)



1975

1949 Joined the CMU faculty

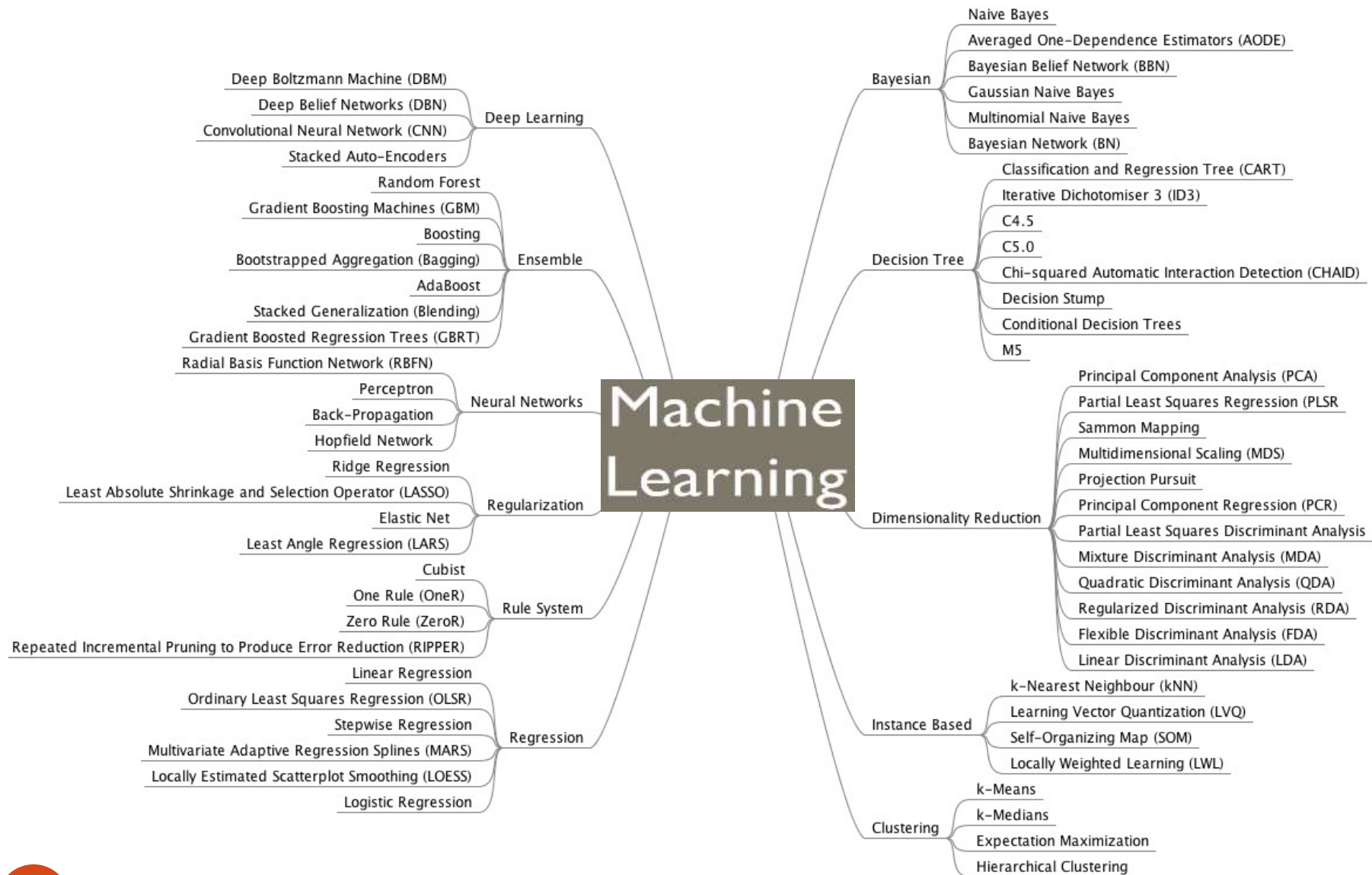
WINNER of the 1978 Nobel Prize in Economics

One of the **MOST INFLUENTIAL SOCIAL SCIENTISTS** of the twentieth century

EXPLORATION OF LEARNING is a common thread across his work and career.

Improvement in **post-secondary education** will require converting teaching from a solo sport to a community based research activity. //

Models and algorithms



Steps

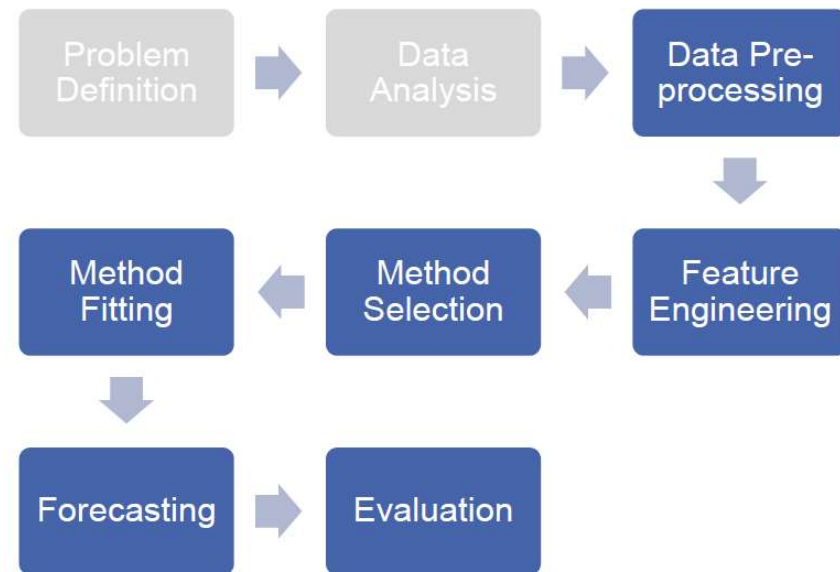
Data pre-processing

Feature engineering

Method selection

Method (model) fitting

Evaluation



Feature engineering

- Feature engineering, also known as **feature creation**, is the process of constructing new features from existing data to train a machine learning model.
- Feature engineering is the process of using domain knowledge of the data to create variables that make machine learning algorithms work.
- Feature engineering starts with retaining existing good features, creating additional useful features and discarding irrelevant features from the dataset.
- Feature engineering is the process of transforming raw data into features that better represent the underlying problem to the predictive models, resulting in improved model accuracy on unseen data (Jason Brownlee).
- Transforming data to create model inputs.

Feature engineering = creating features of the appropriate granularity for the task



<https://www.slideshare.net/gabrielpmoreira/feature-engineering-getting-most-out-of-data-for-predictive-models-tdc-2017>

<https://www.slideshare.net/odsc/feature-engineering>

<https://towardsdatascience.com/four-unavoidable-tips-for-the-art-of-feature-engineering-in-machine-learning-ab4d1ce0cbda>

Feature transformation

Some machine learning models assume that the variables are normally distributed. Other models may benefit from a more homogeneous spread of values across the value range. If variables are not normally distributed, we can apply a mathematical transformation to enforce this distribution.

Typically used mathematical transformations are:

- Logarithm transformation
 - Reciprocal transformation
 - Square/Cubic root transformation
 - Power/Polynomial transformation
 - Exponential transformation
 - Hyperbolic tangent transformation
- $\log(x)$
 - $1 / x$
 - $\text{sqrt}(x)$, $x^{(1/3)}$
 - x^2 , x^3
 - $\exp(x)$
 - $\tanh(x)$
- Compresses the range of large numbers and expand the range of small numbers.

Dealing with **heteroskedasticity**:

- Box-Cox transformation (1964)

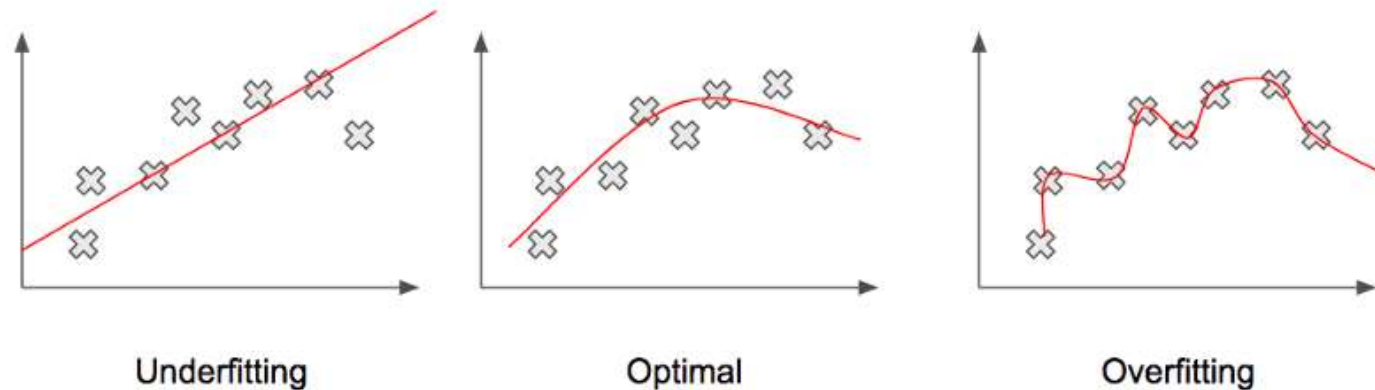
$$y_i^{(\lambda)} = \begin{cases} \frac{y_i^\lambda - 1}{\lambda} & \text{if } \lambda \neq 0, \\ \ln y_i & \text{if } \lambda = 0 \end{cases}$$

$\lambda = 1$ (no transformation)
 $\lambda = 0$ (log)
 $\lambda = 0.5$ (square root)
 $\lambda = -1$ (inverse)

- Yeo-Johnson transformation (2000)

$$y_i^{(\lambda)} = \begin{cases} ((y_i + 1)^\lambda - 1)/\lambda & \text{if } \lambda \neq 0, y \geq 0 \\ \log(y_i + 1) & \text{if } \lambda = 0, y \geq 0 \\ -[(-y_i + 1)^{(2-\lambda)} - 1]/(2 - \lambda) & \text{if } \lambda \neq 2, y < 0 \\ -\log(-y_i + 1) & \text{if } \lambda = 2, y < 0 \end{cases}$$

Model validation and tuning lifecycle



A model is said to be **underfitting** (**High Bias**) when it performs poorly on the training data.

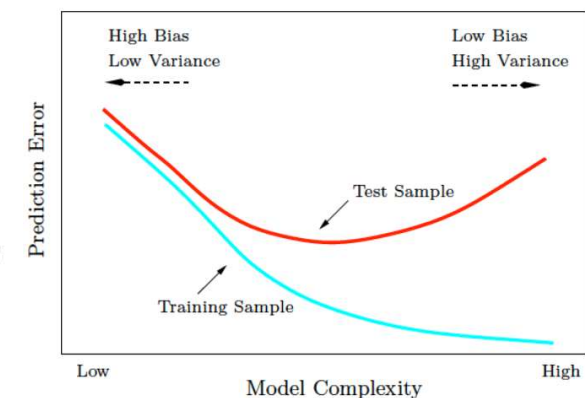
An **overfitting** model, (**High Variance**) on the other hand, performs well on the training data but does not perform well on the evaluation data.

Generally, for a **more complex**/capable model: \downarrow **bias**, \uparrow **variance**

$$E = \underbrace{\text{systematic error}}_{\text{Bias: average prediction's deviation from the truth}} + \underbrace{\text{dependence on specific sample}}_{\text{Variance: sensitivity of prediction to specific training sample}} + \underbrace{\text{random nature of process}}_{\text{Irreducible error or Bayes' rate: due to "noise"}}$$

i.e., due to an under-complex model for a real-world task

i.e., change expected if we use a different training set



Regression/Forecasting models

Linear regression

Polynomial regression

Ridge regression, LASSO regression, Elasticnet

Quantile regression

Support vector regression (SVR)

Nearest neighbor (kNN)

Gaussian process

Decision tree regression

Random forest regression

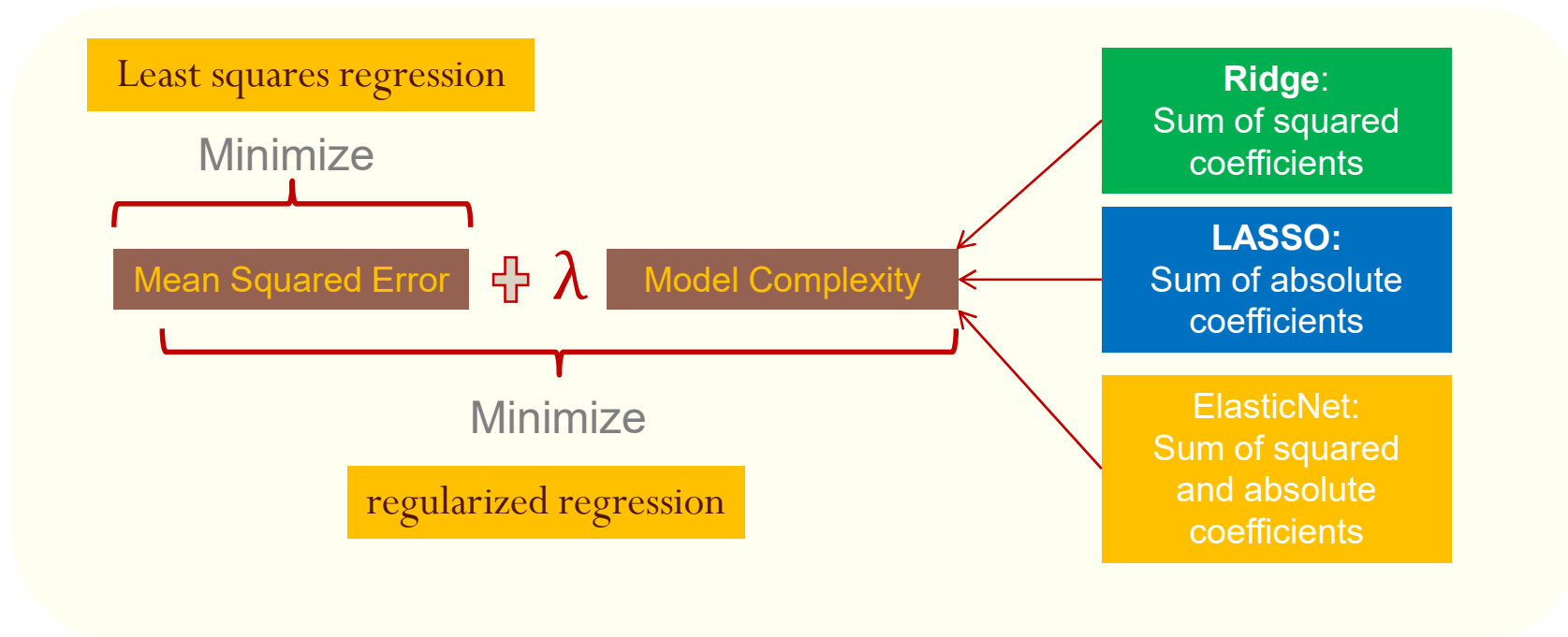
Extra trees regression

Cubist

Gradient boosting machine, CatBoost, XGBoost, LightGBM

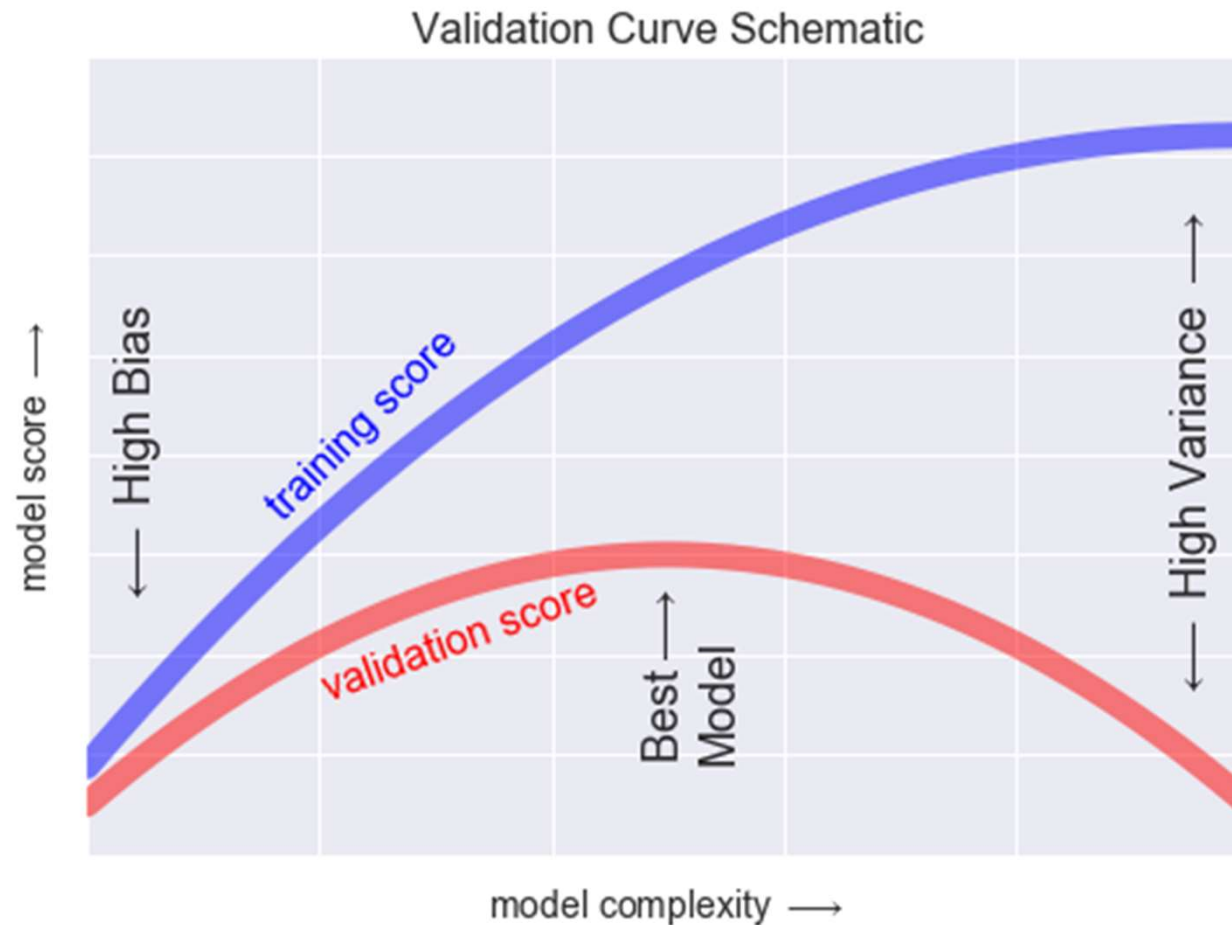
Artificial neural networks (shallow networks and deep learning)

Regularized regression



- ✓ Any regularized regression approach tries to **balance** model performance and model complexity
- ✓ **λ – regularization parameter**, to be estimated
 - $\lambda = \infty$ Null model zero-coefficients (maximum possible penalty)
 - $\lambda = 0$ Least squares solution (no penalty)

Model validation and tuning lifecycle





Performance analysis

Performance analysis



Performance metrics

R-squared (R^2) coefficient of determination

Adjusted R-squared (R^2)

Mean Squared Error (MSE)

Root Mean Squared Error (RMSE)

Mean Absolute Error (MAE)

Mean Absolute Percentage Error (MAPE)

Symmetric Mean Absolute Percentage Error (sMAPE)

Weighted Mean Absolute Percentage Error (WMAPE)

Mean Absolute Scaled Error (MASE)



Cross-validation



Normality tests of the residuals

Shapiro-Walk test

<https://docs.scipy.org/doc/scipy/reference/generated/scipy.stats.shapiro.html>

D'Agostino's K^2 test

<https://docs.scipy.org/doc/scipy/reference/generated/scipy.stats.normaltest.html>

Anderson-Darling test

<https://docs.scipy.org/doc/scipy/reference/generated/scipy.stats.anderson.html>

Performance analysis

Regression diagnostics and specification tests

✓ Heteroscedasticity tests

For these test the null hypothesis is that all observations have the same error variance, i.e. errors are homoscedastic. The tests differ in which kind of heteroscedasticity is considered as alternative hypothesis.

✓ Autocorrelation tests

This group of test whether the regression residuals are not autocorrelated. They assume that observations are ordered by time.

✓ Non-linearity tests

✓ Tests for Structural change, Parameter stability

Test whether all or some regression coefficient are constant over the entire data sample.

✓ Multicollinearity tests



✓ Normality and distribution tests

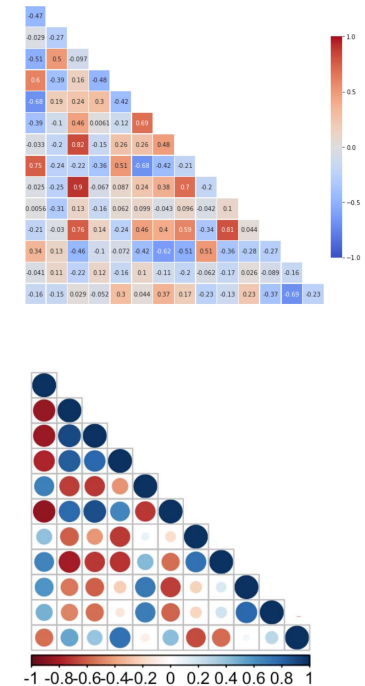
Multicollinearity

Correlation matrix / Correlation plot is used to detect the presence of multicollinearity. A correlation plot can be used to identify the correlation or bivariate relationship between two independent variables whereas **Variation Inflation Factor (VIF)** is used to identify the correlation of one independent variable with a group of other variables.

Hence, it is preferred to use **VIF** for better understanding.

VIF = 1 → No correlation
VIF = 1 to 5 → Moderate correlation
VIF > 10 → High correlation

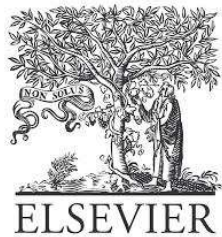
$$VIF = \frac{1}{(1-R^2)}$$



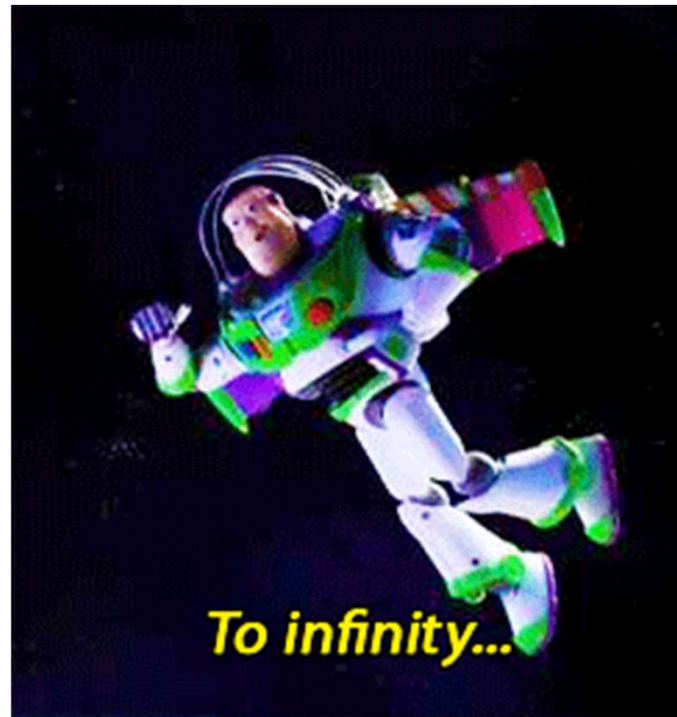


<https://bdtb.ibict.br/vufind/>

References

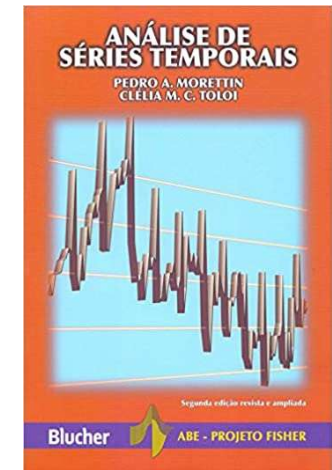
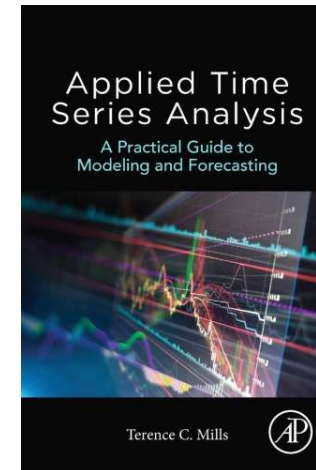
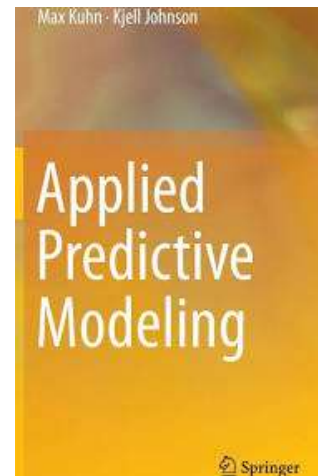
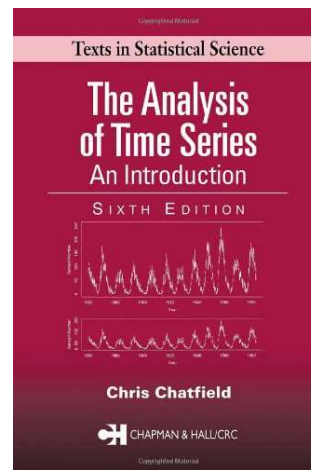
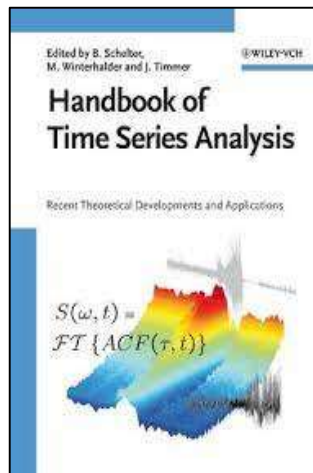
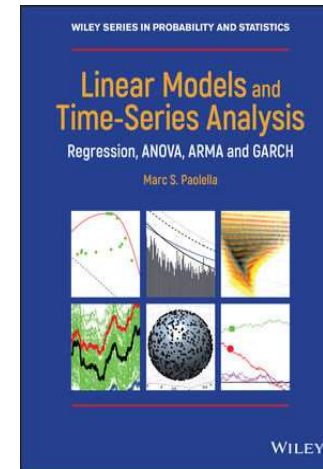
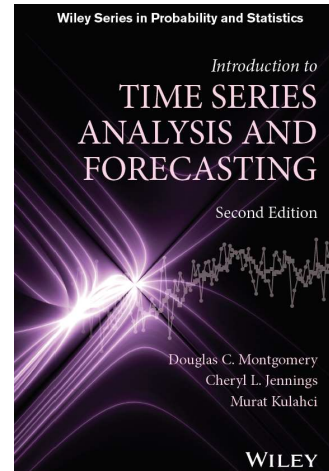
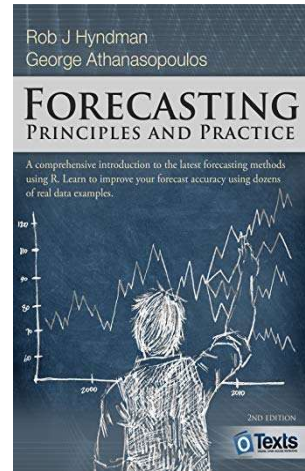
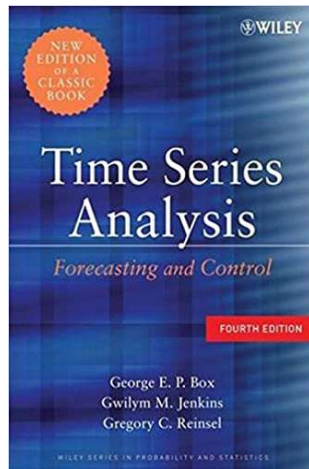


To be continued ...



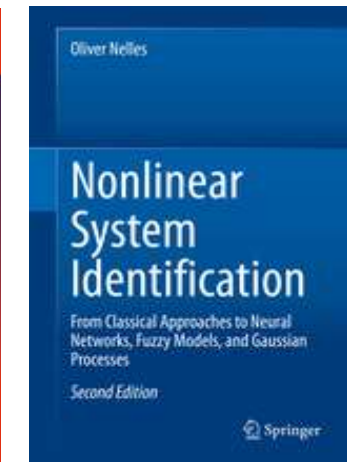
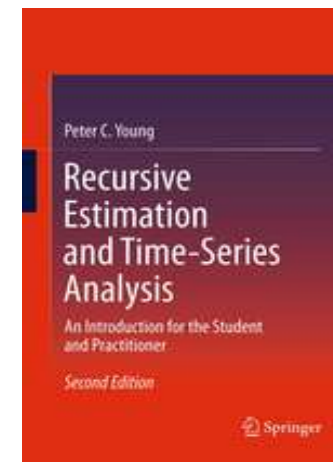
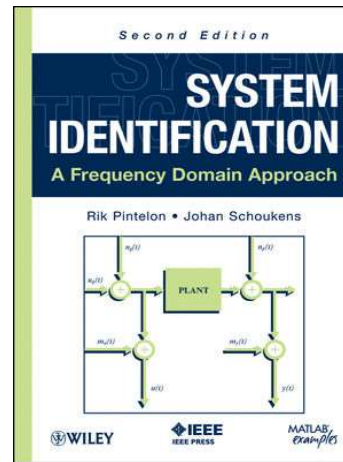
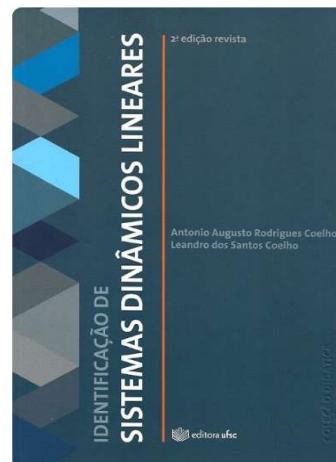
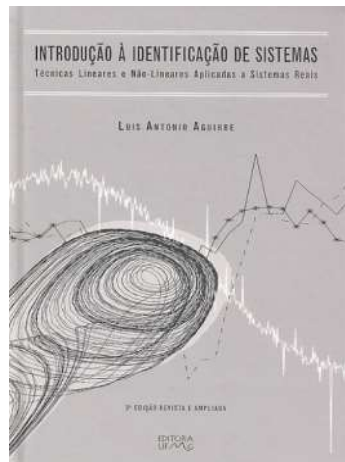
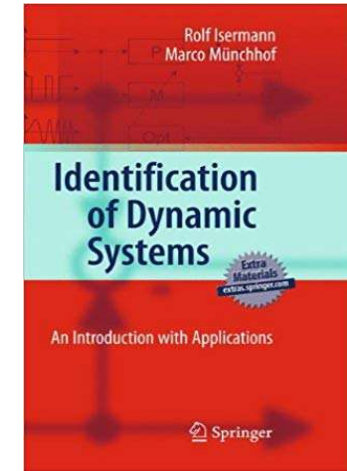
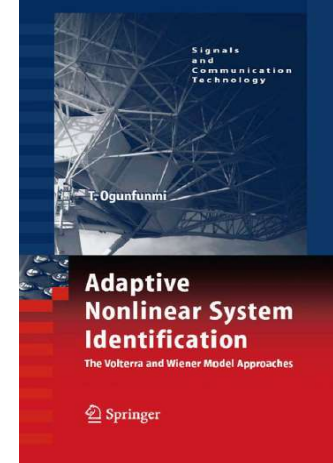
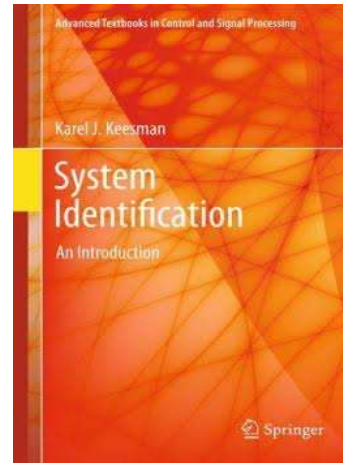
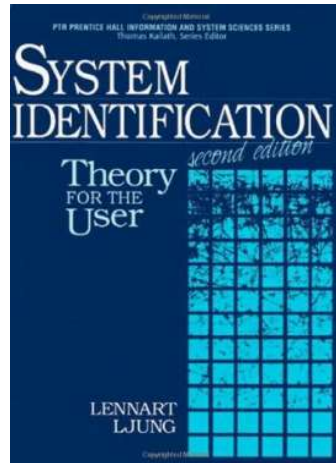
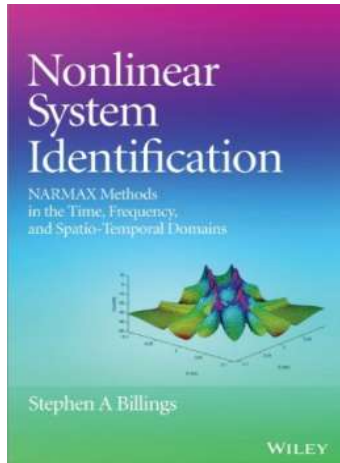
Books

Time series forecasting



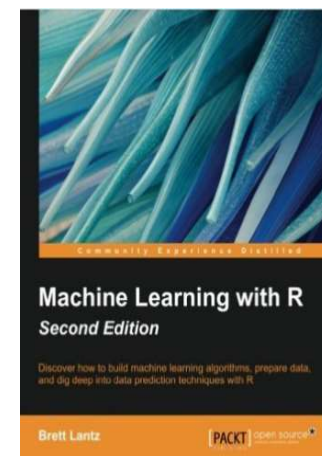
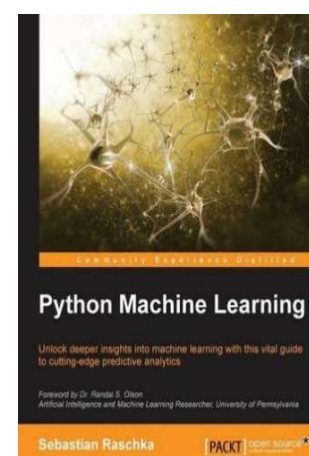
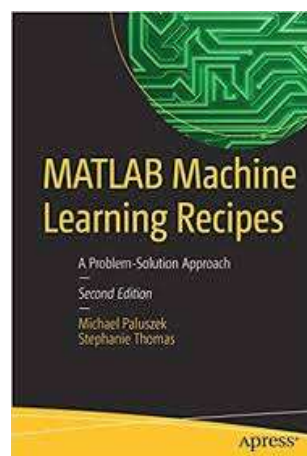
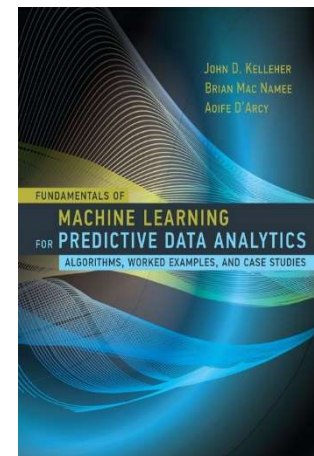
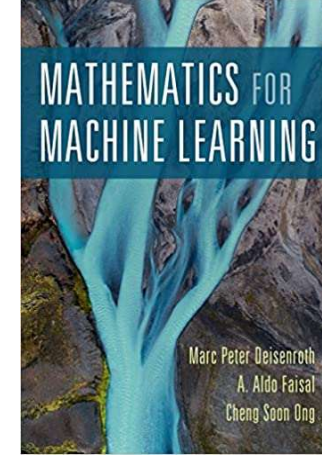
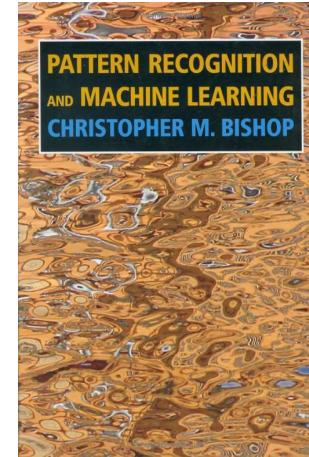
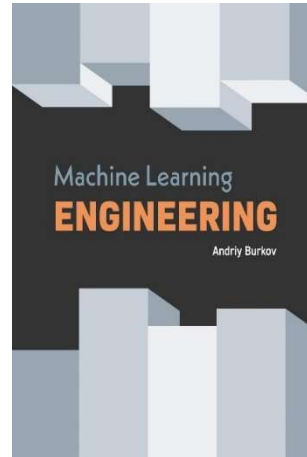
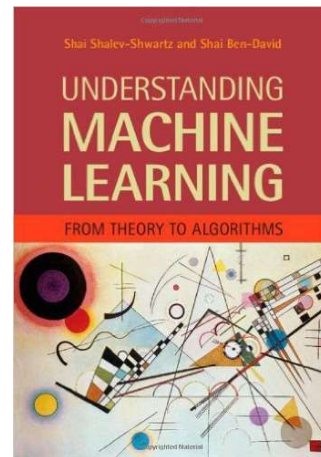
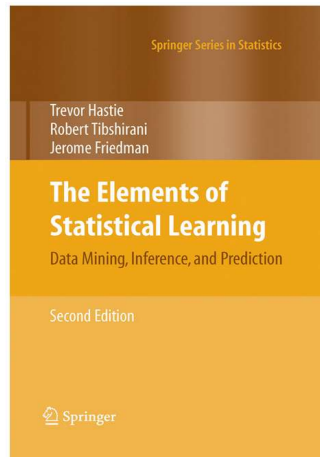
Books

System identification

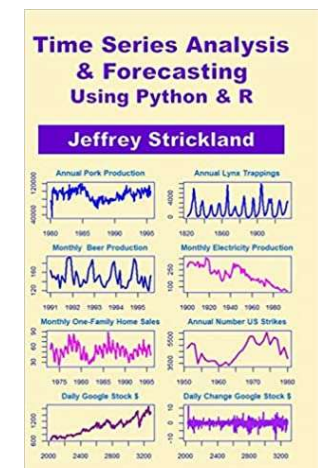
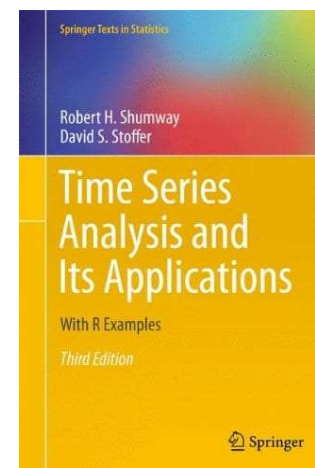
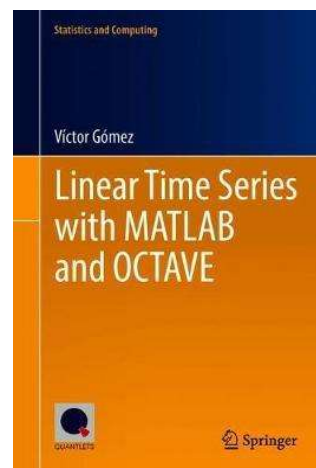
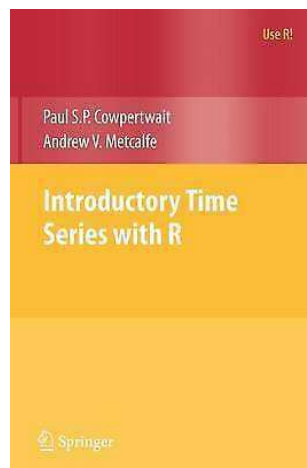
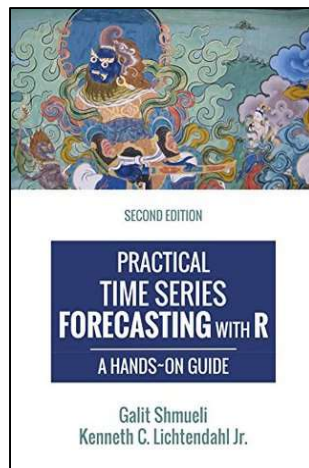
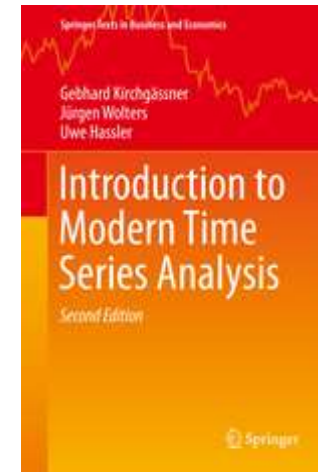
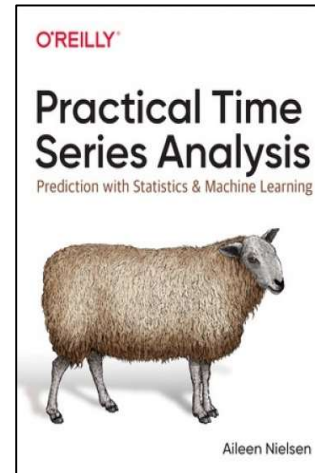
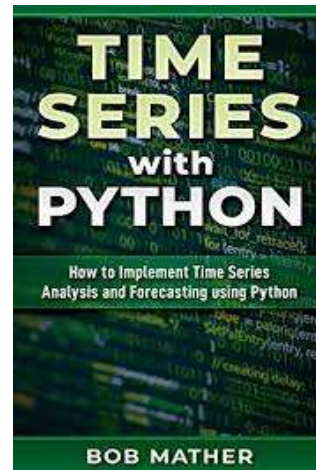
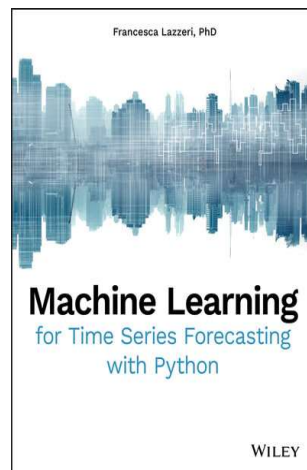
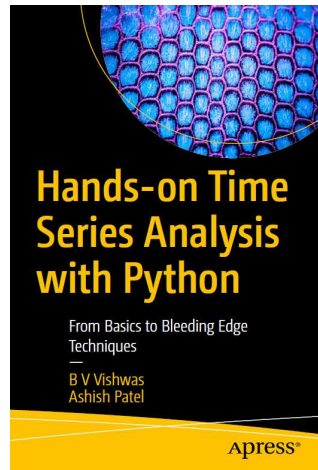
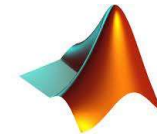


Books

Machine learning



Books



Quote

If time travel is possible, **where are** the tourists from the future?

Stephen William Hawking (1942-2018)

English theoretical physicist, cosmologist, and author who was director of research at the Centre for Theoretical Cosmology at the University of Cambridge at the time of his death.

It is a curious fact that Stephen William Hawking was born on 8th January 1942, exactly **300** years after the death of the Italian astronomer, Galileo Galilei.





III Webinar 2021

26/08/2021 , 17h30

CT de Identificação de Sistemas e Ciência de Dados



<https://www.youtube.com/watch?v=5bB6hOcFSJQ>



Análise e previsão de séries temporais

Time series analysis and forecasting

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